Precision Measurements of the Top Quark Mass at the Tevatron

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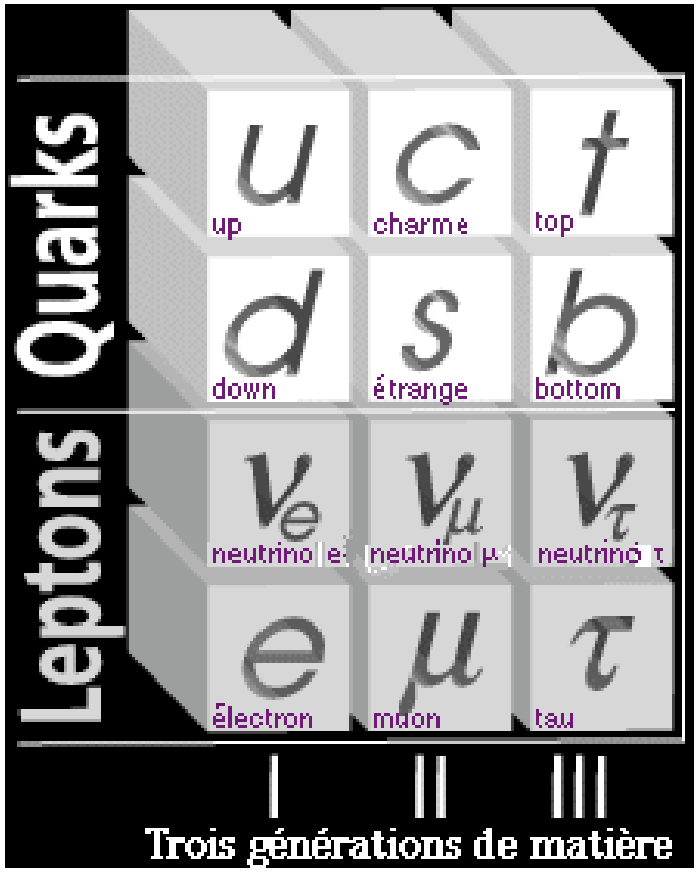
On behalf of the CDF and DZero Collaborations

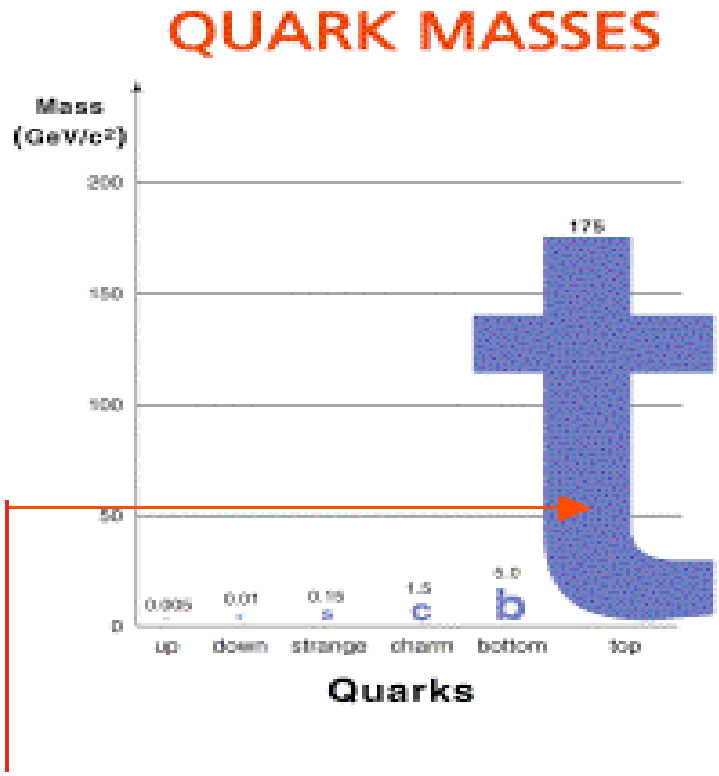
The Standard Model

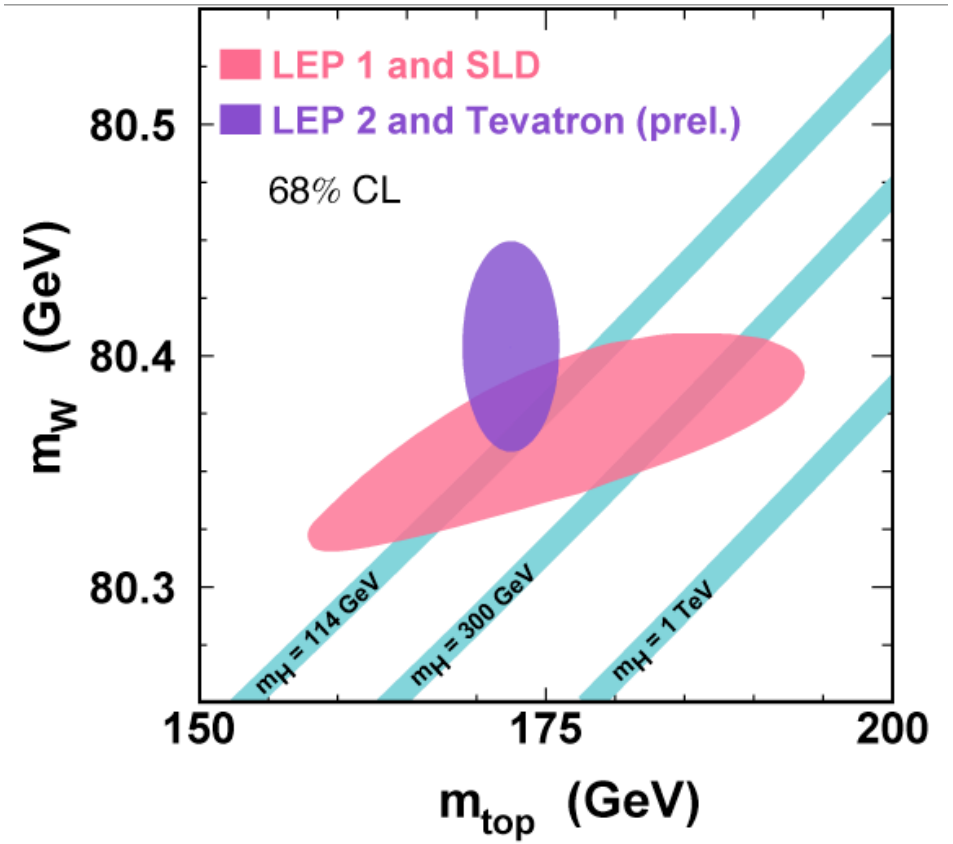


The Standard Model

$$
\begin{gathered}
-\frac{1}{2}\partial_\nu g^a_\mu \partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \frac{1}{4}g_s^2 f^{abc} f^{ade} g^b_\mu g^c_\nu g^d_\mu g^e_\nu + \\
\frac{1}{2}ig_s^2(\bar{q}^\sigma_i \gamma^\mu q^\sigma_j) g^a_\mu + \bar{G}^a \partial^2 G^a + g_s f^{abc}\partial_\mu \bar{G}^a G^b g^c_\mu - \partial_\nu W^+_\mu \partial_\nu W^-_\mu - \\
M^2 W^+_\mu W^-_\mu - \frac{1}{2}\partial_\nu Z^0_\mu \partial_\nu Z^0_\mu - \frac{1}{2c_w^2} M^2 Z^0_\mu Z^0_\mu - \frac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - \frac{1}{2}\partial_\mu H \partial_\mu H - \\
\frac{1}{2}m_h^2 H^2 - \partial_\mu \phi^+ \partial_\mu \phi^- - M^2 \phi^+ \phi^- - \frac{1}{2}\partial_\mu \phi^0 \partial_\mu \phi^0 - \frac{1}{2c_w^2} M \phi^0 \phi^0 - \beta_h [\frac{2M^2}{g^2} + \\
\frac{2M}{g} H + \frac{1}{2}(H^2 + \phi^0 \phi^0 + 2\phi^+ \phi^-)] + \frac{2M^4}{g^2}\alpha_h - igc_w[\partial_\nu Z^0_\mu (W^+_\mu W^-_\nu - \\
W^+_\nu W^-_\mu) - Z^0_\nu (W^+_\mu \partial_\nu W^-_\mu - W^-_\mu \partial_\nu W^+_\mu) + Z^0_\mu (W^+_\nu \partial_\nu W^-_\mu - \\
W^-_\nu \partial_\nu W^+_\mu)] - igs_w[\partial_\nu A_\mu (W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - A_\nu (W^+_\mu \partial_\nu W^-_\mu - \\
W^-_\mu \partial_\nu W^+_\mu) + A_\mu (W^+_\nu \partial_\nu W^-_\mu - W^-_\nu \partial_\nu W^+_\mu)] - \frac{1}{2}g^2 W^+_\mu W^-_\mu W^+_\nu W^-_\nu + \\
\frac{1}{2}g^2 W^+_\mu W^-_\nu W^+_\mu W^-_\nu + g^2 c_w^2 (Z^0_\mu W^+_\mu Z^0_\nu W^-_\nu - Z^0_\mu Z^0_\mu W^+_\nu W^-_\nu) + \\
g^2 s_w^2 (A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu) + g^2 s_w c_w [A_\mu Z^0_\nu (W^+_\mu W^-_\nu - \\
W^+_\nu W^-_\mu) - 2A_\mu Z^0_\mu W^+_\nu W^-_\nu] - g\alpha[H^3 + H\phi^0\phi^0 + 2H\phi^+\phi^-] - \\
\frac{1}{8}g^2\alpha_h[H^4 + (\phi^0)^4 + 4(\phi^+\phi^-)^2 + 4(\phi^0)^2\phi^+\phi^- + 4H^2\phi^+\phi^- + 2(\phi^0)^2 H^2] - \\
gMW^+_\mu W^-_\mu H - \frac{1}{2}g\frac{M}{c_w^2} Z^0_\mu Z^0_\mu H - \frac{1}{2}ig[W^+_\mu(\phi^0\partial_\mu\phi^- - \phi^-\partial_\mu\phi^0) - \\
W^-_\mu(\phi^0\partial_\mu\phi^+ - \phi^+\partial_\mu\phi^0)] + \frac{1}{2}g[W^+_\mu(H\partial_\mu\phi^- - \phi^-\partial_\mu H) - W^-_\mu(H\partial_\mu\phi^+ - \\
\phi^+\partial_\mu H)] + \frac{1}{2}g\frac{1}{c_w}(Z^0_\mu(H\partial_\mu\phi^0 - \phi^0\partial_\mu H) - ig\frac{s_w^2}{c_w} M Z^0_\mu (W^+_\mu\phi^- - W^-_\mu\phi^+) + \\
igs_w M A_\mu (W^+_\mu \phi^- - W^-_\mu \phi^+) - ig\frac{1-2c_w^2}{2c_w} Z^0_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) + \\
igs_w A_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) - \frac{1}{4}g^2 W^+_\mu W^-_\mu [H^2 + (\phi^0)^2 + 2\phi^+\phi^-] - \\
\frac{1}{4}g^2\frac{1}{c_w^2} Z^0_\mu Z^0_\mu [H^2 + (\phi^0)^2 + 2(2s_w^2 - 1)^2\phi^+\phi^-] - \frac{1}{2}g^2\frac{s_w^2}{c_w} Z^0_\mu \phi^0 (W^+_\mu \phi^- + \\
W^-_\mu \phi^+) - \frac{1}{2}ig^2\frac{s_w^2}{c_w} Z^0_\mu H (W^+_\mu \phi^- - W^-_\mu \phi^+) + \frac{1}{2}g^2 s_w A_\mu \phi^0 (W^+_\mu \phi^- + \\
W^-_\mu \phi^+) + \frac{1}{2}ig^2 s_w A_\mu H (W^+_\mu \phi^- - W^-_\mu \phi^+) - g^2\frac{s_w}{c_w}(2c_w^2 - 1) Z^0_\mu A_\mu \phi^+\phi^- - \\
g^1 s_w^2 A_\mu A_\mu \phi^+\phi^- - \bar{e}^\lambda(\gamma\partial + m_e^\lambda)e^\lambda - \bar{\nu}^\lambda\gamma\partial\nu^\lambda - \bar{u}^\lambda_j(\gamma\partial + m_u^\lambda)u^\lambda_j - \\
\bar{d}^\lambda_j(\gamma\partial + m_d^\lambda)d^\lambda_j + igs_w A_\mu[-(\bar{e}^\lambda\gamma^\mu e^\lambda) + \frac{2}{3}(\bar{u}^\lambda_j\gamma^\mu u^\lambda_j) - \frac{1}{3}(\bar{d}^\lambda_j\gamma^\mu d^\lambda_j)] + \\
\frac{ig}{4c_w} Z^0_\mu [(\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar{e}^\lambda\gamma^\mu(4s_w^2 - 1 - \gamma^5)e^\lambda) + (\bar{u}^\lambda_j\gamma^\mu(\frac{4}{3}s_w^2 - \\
1 - \gamma^5)u^\lambda_j) + (\bar{d}^\lambda_j\gamma^\mu(1 - \frac{8}{3}s_w^2 - \gamma^5)d^\lambda_j)] + \frac{ig}{2\sqrt{2}} W^+_\mu [(\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5)e^\lambda) + \\
(\bar{u}^\lambda_j\gamma^\mu(1+\gamma^5)C_{\lambda\kappa}d^\kappa_j)] + \frac{ig}{2\sqrt{2}} W^-_\mu [(\bar{e}^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar{d}^\kappa_j C^\dagger_{\lambda\kappa}\gamma^\mu(1 + \\
\gamma^5)u^\lambda_j)] + \frac{ig}{2\sqrt{2}}\frac{m_e^\lambda}{M}[-\phi^+(\bar{\nu}^\lambda(1-\gamma^5)e^\lambda) + \phi^-(\bar{e}^\lambda(1+\gamma^5)\nu^\lambda)] - \\
\frac{g}{2}\frac{m_e^\lambda}{M}[H(\bar{e}^\lambda e^\lambda) + i\phi^0(\bar{e}^\lambda\gamma^5 e^\lambda)] + \frac{ig}{2M\sqrt{2}}\phi^+[-m_d^\kappa(\bar{u}^\lambda_j C_{\lambda\kappa}(1-\gamma^5)d^\kappa_j) + \\
m_u^\lambda(\bar{u}^\lambda_j C_{\lambda\kappa}(1+\gamma^5)d^\kappa_j] + \frac{ig}{2M\sqrt{2}}\phi^-[m_d^\lambda(\bar{d}^\lambda_j C^\dagger_{\lambda\kappa}(1+\gamma^5)u^\kappa_j) - m_u^\kappa(\bar{d}^\lambda_j C^\dagger_{\lambda\kappa}(1 - \\
\gamma^5)u^\kappa_j] - \frac{g}{2}\frac{m_u^\lambda}{M}H(\bar{u}^\lambda_j u^\lambda_j) - \frac{g}{2}\frac{m_d^\lambda}{M}H(\bar{d}^\lambda_j d^\lambda_j) + \frac{ig}{2}\frac{m_u^\lambda}{M}\phi^0(\bar{u}^\lambda_j\gamma^5 u^\lambda_j) - \\
\frac{ig}{2}\frac{m_d^\lambda}{M}\phi^0(\bar{d}^\lambda_j\gamma^5 d^\lambda_j) + \bar{X}^+(\partial^2 - M^2)X^+ + \bar{X}^-(\partial^2 - M^2)X^- + \bar{X}^0(\partial^2 - \\
\frac{M^2}{c_w^2})X^0 + \bar{Y}\partial^2 Y + igc_w W^+_\mu(\partial_\mu\bar{X}^0 X^- - \partial_\mu\bar{X}^+ X^0) + igs_w W^+_\mu(\partial_\mu\bar{Y}X^- - \\
\partial_\mu\bar{X}^+ Y) + igc_w W^-_\mu(\partial_\mu\bar{X}^- X^0 - \partial_\mu\bar{X}^0 X^+) + igs_w W^-_\mu(\partial_\mu\bar{X}^- Y - \\
\partial_\mu\bar{Y}X^+) + igc_w Z^0_\mu(\partial_\mu\bar{X}^+ X^+ - \partial_\mu\bar{X}^- X^-) + igs_w A_\mu(\partial_\mu\bar{X}^+ X^+ - \\
\partial_\mu\bar{X}^- X^-) - \frac{1}{2}gM[\bar{X}^+ X^+ H + \bar{X}^- X^- H + \frac{1}{c_w^2}\bar{X}^0 X^0 H] + \\
\frac{1-2c_w^2}{2c_w} igM[\bar{X}^+ X^0 \phi^+ - \bar{X}^- X^0 \phi^-] + \frac{1}{2c_w} igM[\bar{X}^0 X^- \phi^+ - \bar{X}^0 X^+ \phi^-] + \\
igMs_w[\bar{X}^0 X^- \phi^+ - \bar{X}^0 X^+ \phi^-] + \frac{1}{2}igM[\bar{X}^+ X^+ \phi^0 - \bar{X}^- X^- \phi^0]
\end{gathered}
$$

Many Questions about the Top Quark





Quark mass structure

What does it reveal?

Why is the top so heavy?

Why is its Yukawa coupling ~ 1 ?

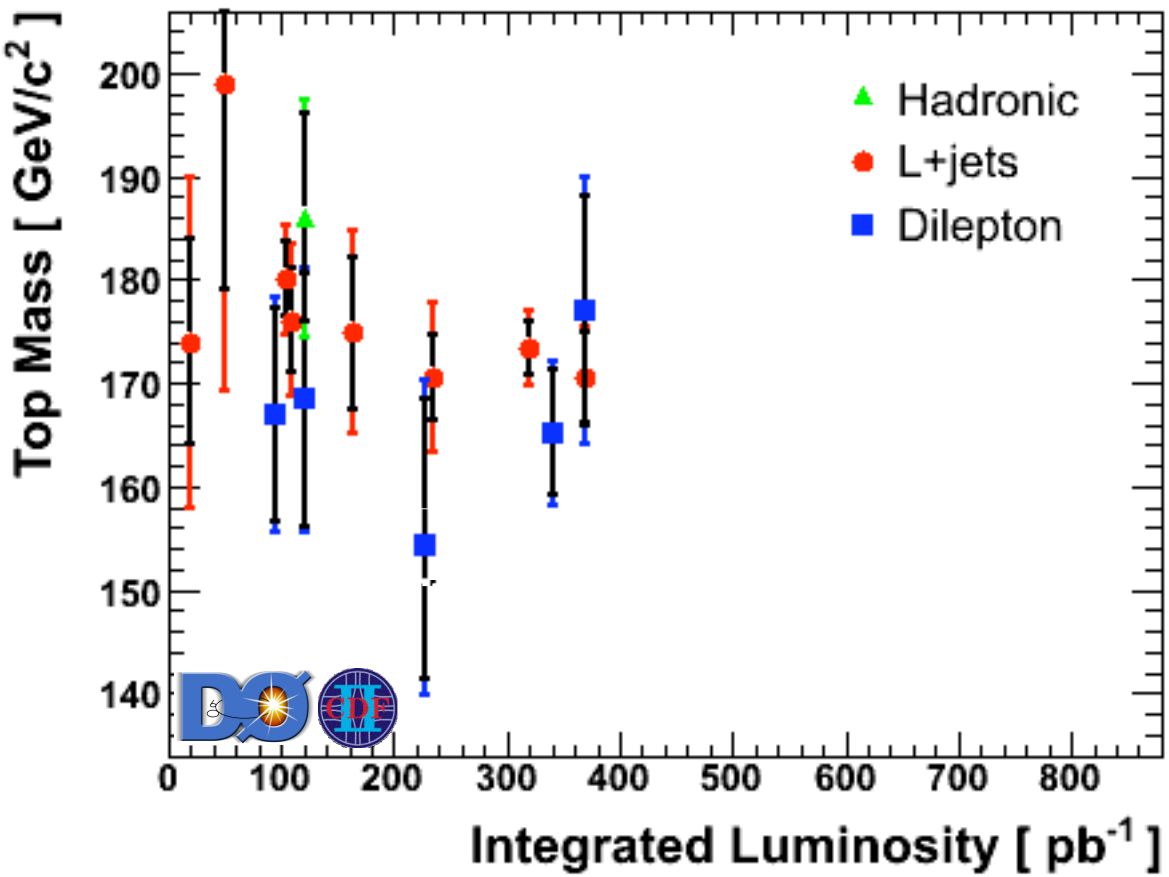
Higgs connection

Radiative corrections

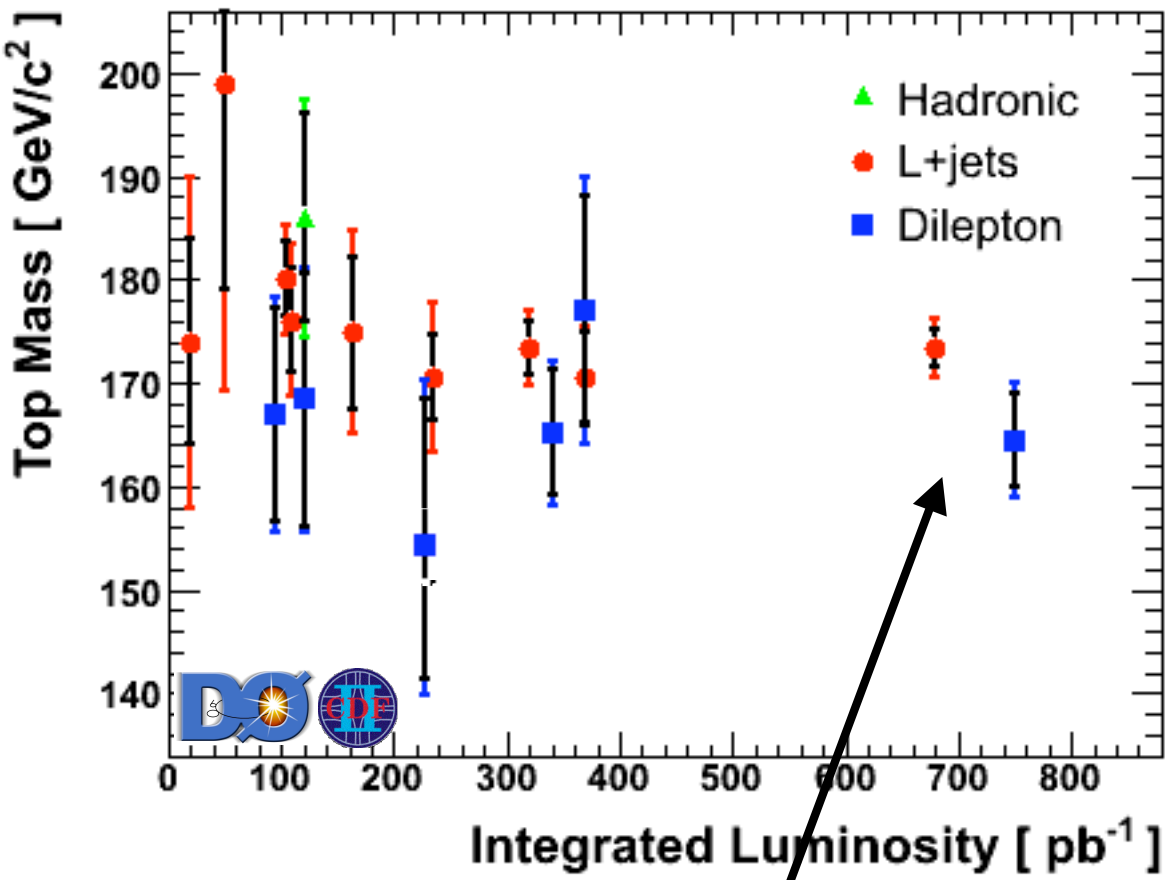
Heavy top means heavy Higgs

M_t provides constraints on M_h

Direct top mass measurements



Direct top mass measurements



L+jets error dominated by systematics!

Dilepton error will soon be systematics limited.

Precision hadronic measurements coming.

M_{top} in 1 fb^{-1}: this summer from D0 and CDF

This talk

- most precise measurement
- channel comparisons
- new world average
- prospects for reducing systematics

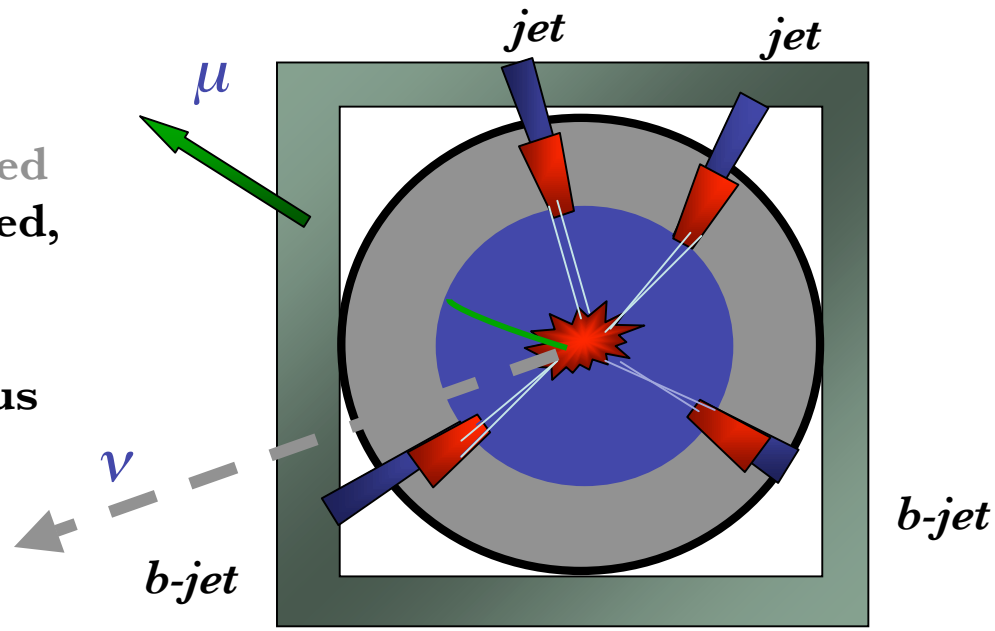
How to measure mass

How do we measure the mass?

Can't just put the pieces back together again

Lost information

- Neutrinos have escaped undetected
- Quarks have hadronized, showered, been clustered into *jets*
- Assignment of reconstructed objects to partons is not obvious
- Lepton resolution is good, but not perfect



Misinformation

- Background processes mimic top-ology

Mass Measurement Technique

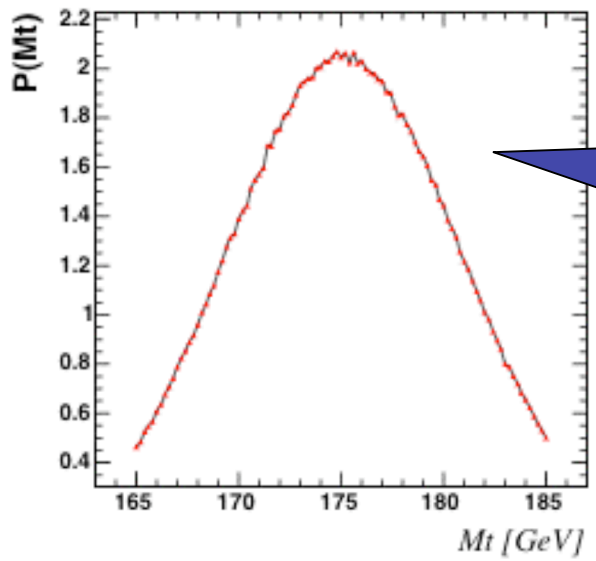
Monte Carlo integration over resolutions & unmeasured quantities

Lepton+jets

4 jet energy resolutions
jet-parton assignments
12 (no *b*-tags)
6 (1 *b*-tag)
2 (2 *b*-tags)
1 missing neutrino

Dileptons

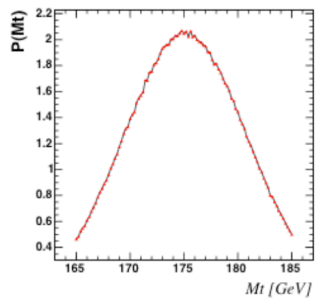
2 jet energy resolutions
2 jet-parton assignments
2 missing neutrinos



Each event has a curve, rather than a single mass value

Construct probability curve $P(M_t)$ for each event.

Probability calculation



Template strategy

Example: parameterize $P(M_t)$ in terms of event-level variable: Reconstucted M_{top}

Derive parameters from templates of simulated events.

Currently, best measurement in *lepton+jets*

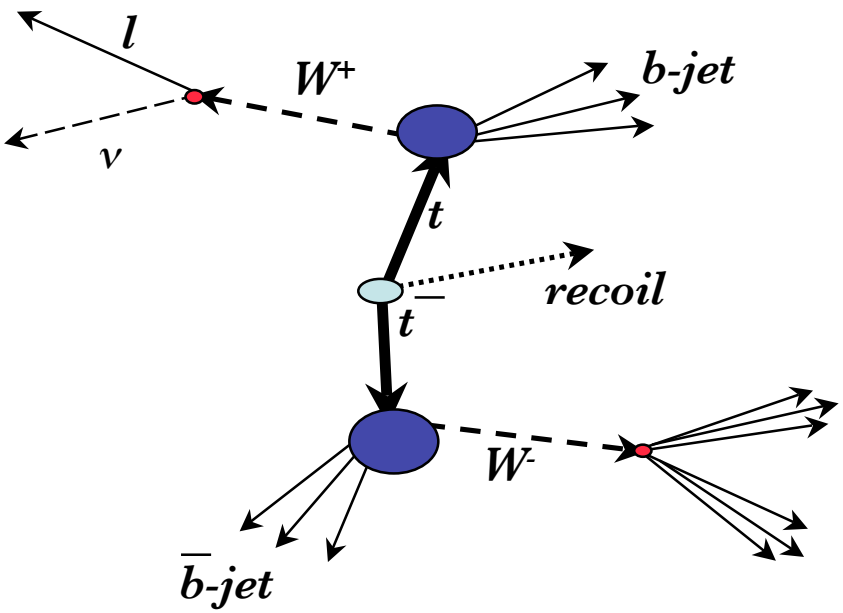
Matrix-element strategy

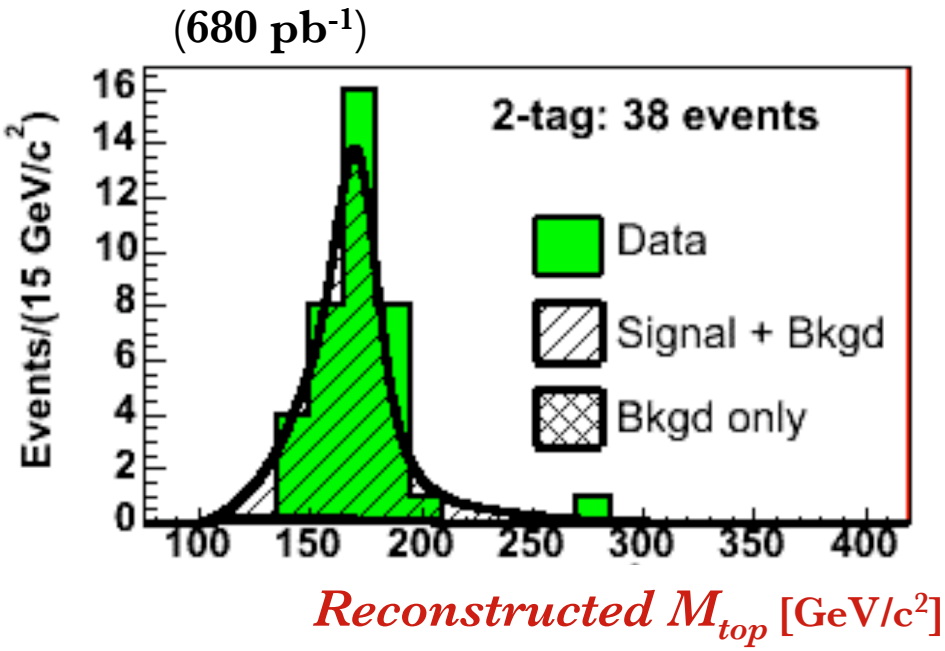
Calculate $P(M_t)$ directly for each event by convoluting matrix element with resolution functions.

Make simplifying approximations and measure corrections in simulated events.

Currently, best measurement in *dileptons*.

CDF Lepton+Jets template





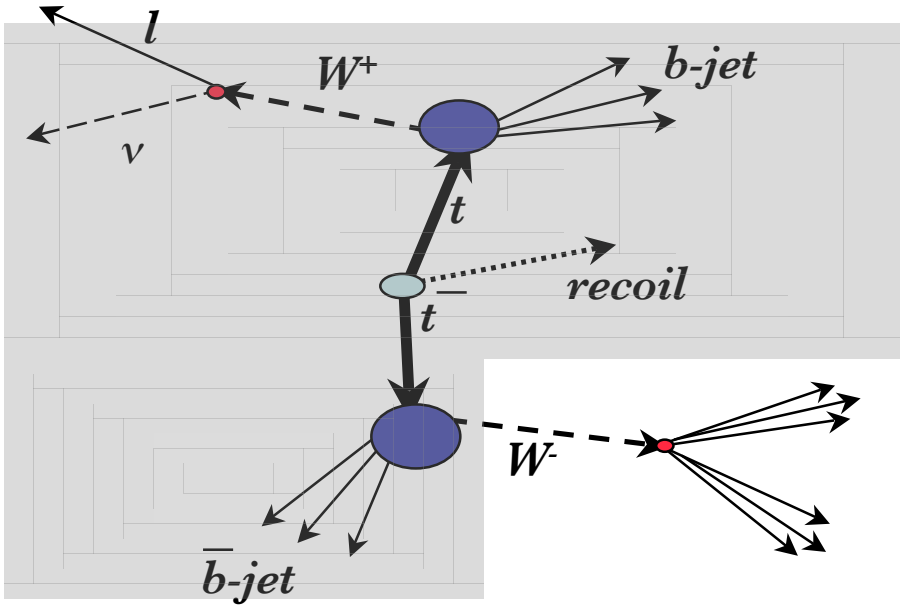
Use assignments with lowest $\chi 2$ to reconstruct top mass.

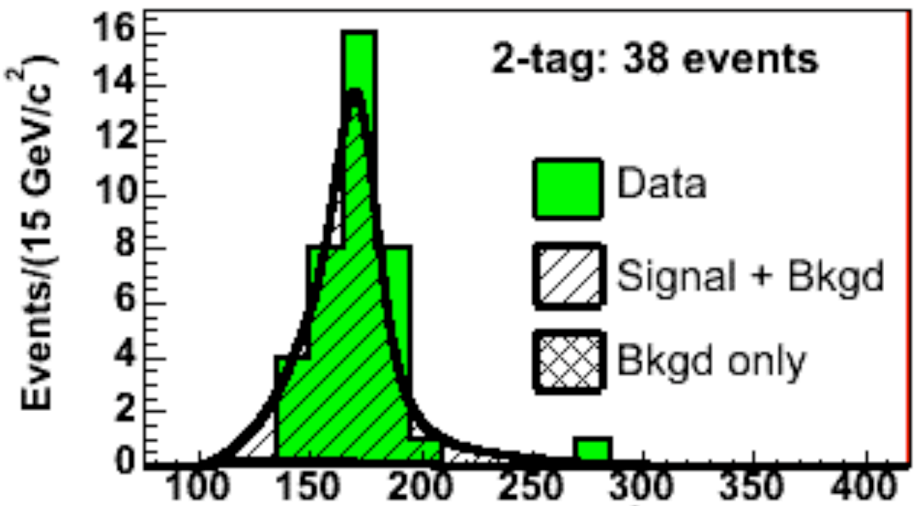
Prefer

- top masses equal

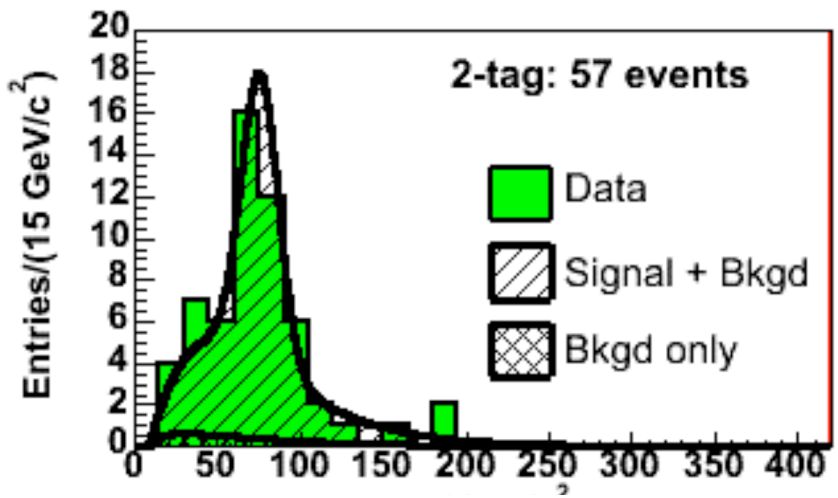
- reconstructed Ws near M_W

CDF Lepton+Jets template





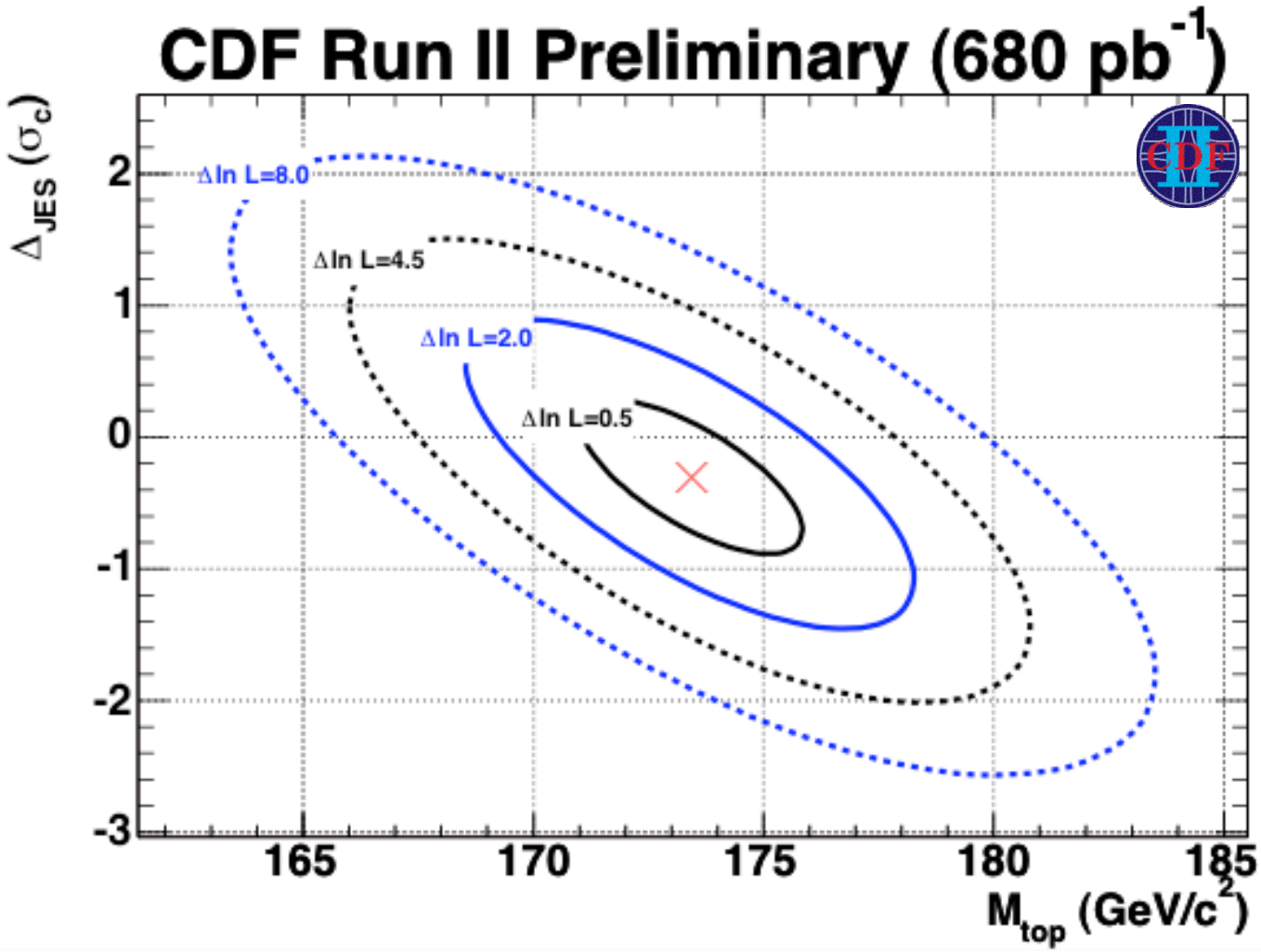
Reconstructed M_{top} [GeV/c²]



Reconstructed W mass [GeV/c²]

Reconstructed *W* mass to calibrate jet energy scale, expressed in units of $\sigma(P_T,\eta)$ from the external calibration.

2D Likelihood



2D Result: $M_t = 173.4 \pm 1.7_{(Stat)} \pm 1.8_{(JES)} \pm 1.3_{(syst)}$ GeV/c^2

$\Delta JES = -0.3 \pm 0.6\ \sigma$

Single best measurement of M_t. More precise than Summer 2005 World Average!

CDF Lepton+Jets: Systematics

Systematics are largely due to uncertainties in modeling.

Source	$\Delta M_{top}(GeV/c^2)$
Residual Jet Energy Scale	0.7
ISR/FSR	0.6
B-jet modeling	0.6
Background Shape	0.5
Background JES	0.4
Parton Distributions	0.3
Generator	0.2
Simulation Statistics	0.3
B-tagging	0.1
Total	1.3

CDF Lepton+Jets: Systematics

Systematics are largely due to uncertainties in modeling.

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Total	1.3

From p_T and η dependence of modeling uncertainties

CDF Lepton+Jets: Systematics

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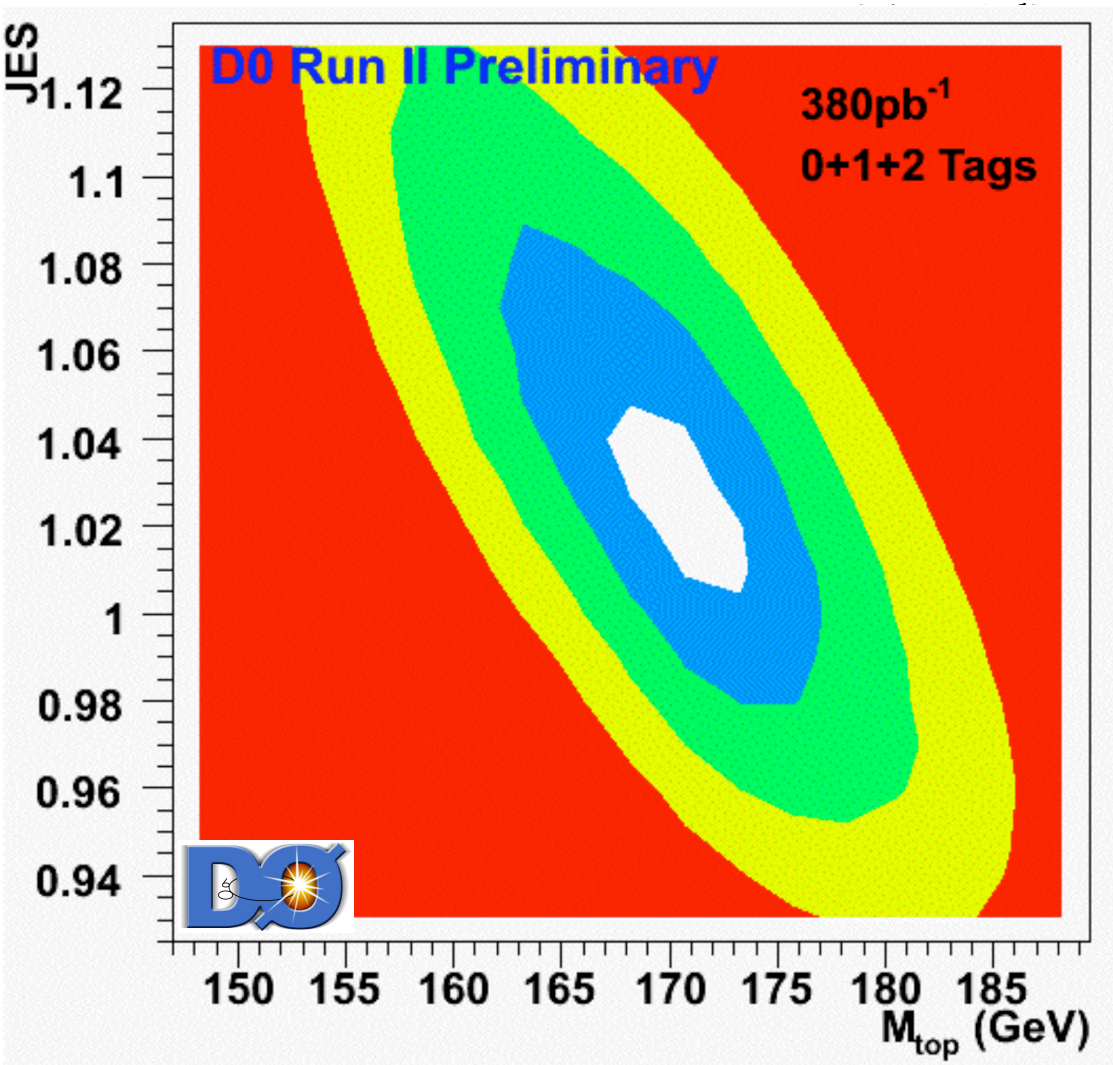
Model constrained by Z+jets data

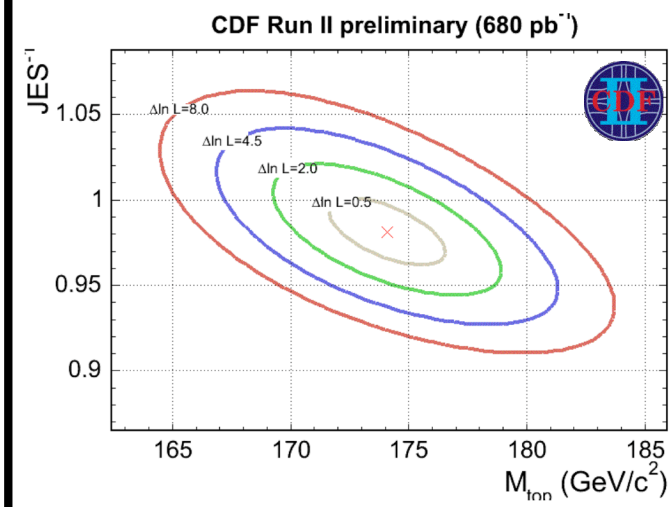
L+jets results



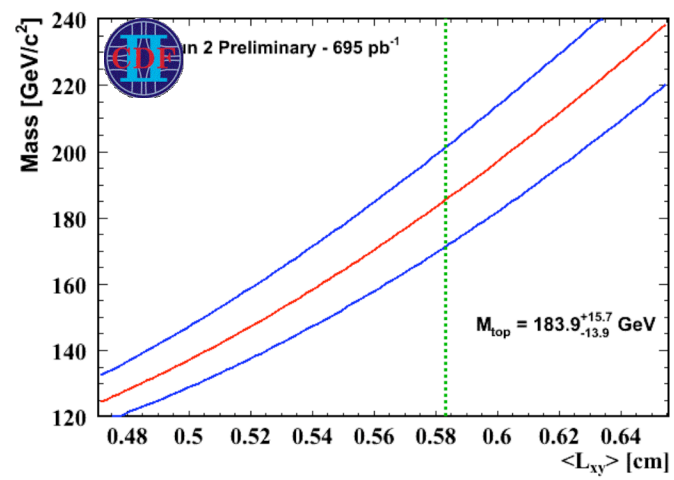
$M_t = 170.6 \pm 4.4_{stat} \pm 1.7_{syst}$

$JES = 1.03 \pm 0.03$

CDF Matrix Element



CDF Transverse Decay Length



$M_t = 183.9 \pm 14.8_{sta} \pm 5.6_{syst}$

L+jets summary

Many methods being used

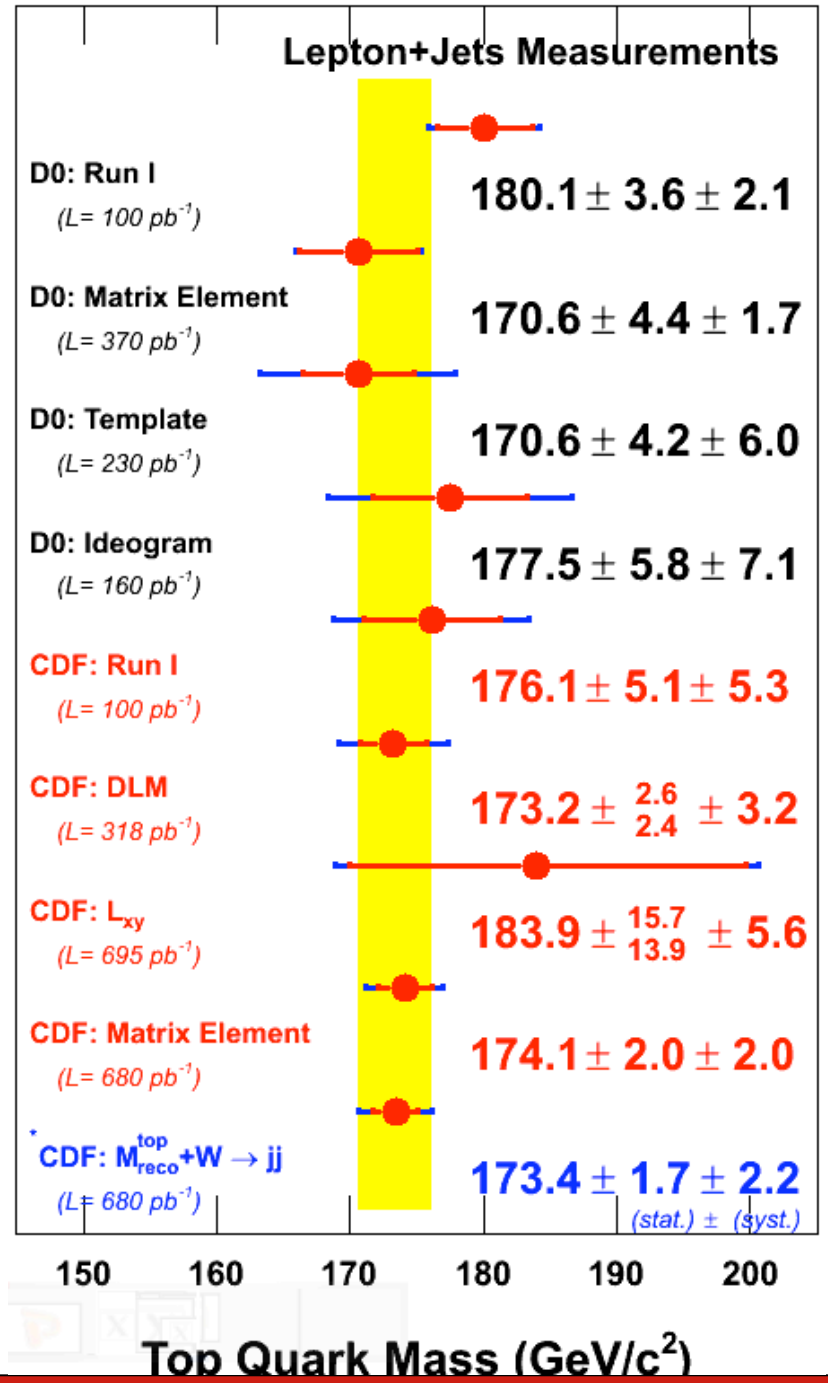
Differing systematic sensitivities

All measurements consistent.

NEW!

NEW!

NEW!

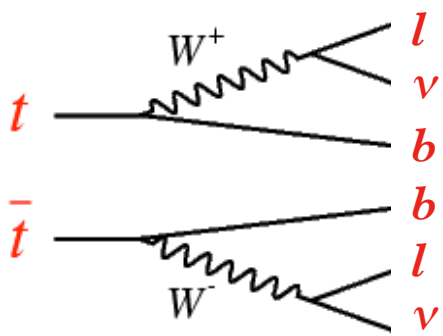


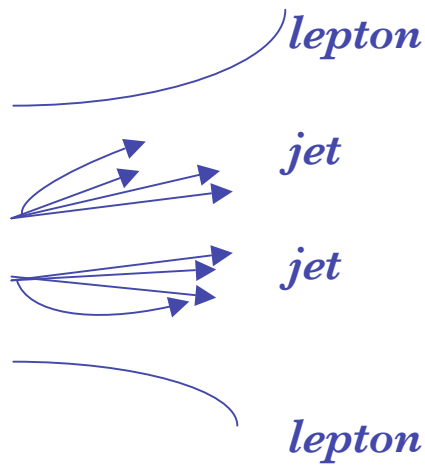
Matrix Element Method

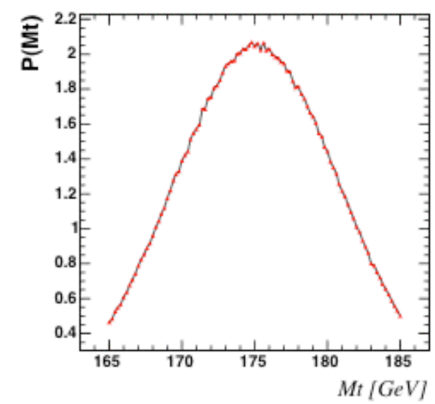
(as applied to dileptons at CDF)

Break into two pieces: parton-level process and showering/resolution effects

$$P(\text{partons} \mid M_t) \quad x \quad P(\text{event } x \mid \text{partons}) \quad = \quad P(\text{event } x \mid M_t)$$







Assume

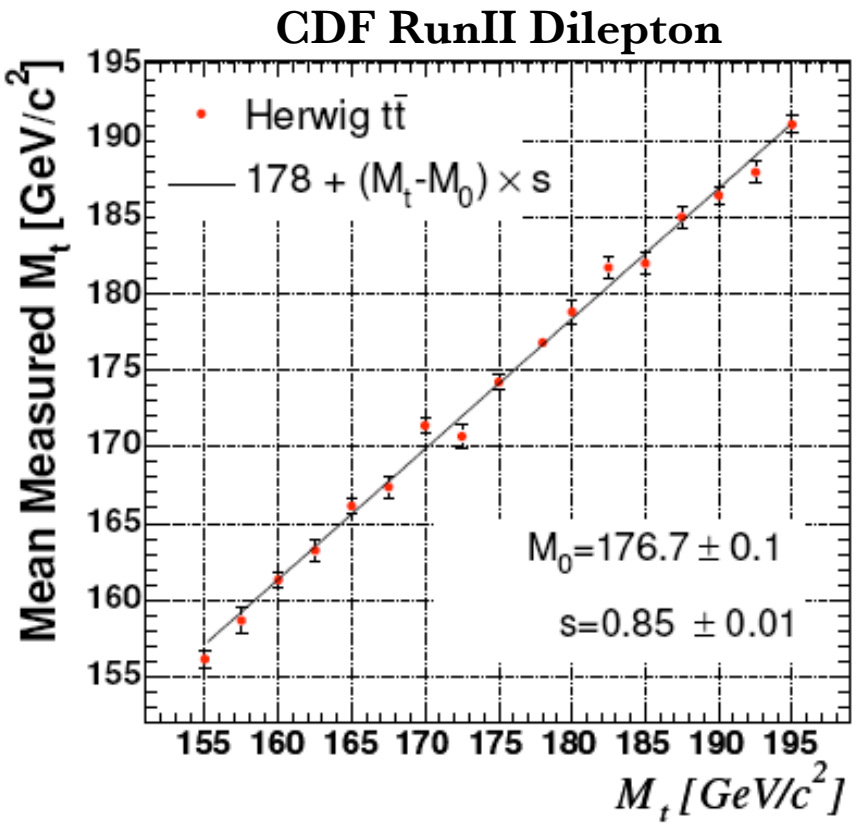
Jet angles are perfect
Leptons are perfect
Jets are *b*'s

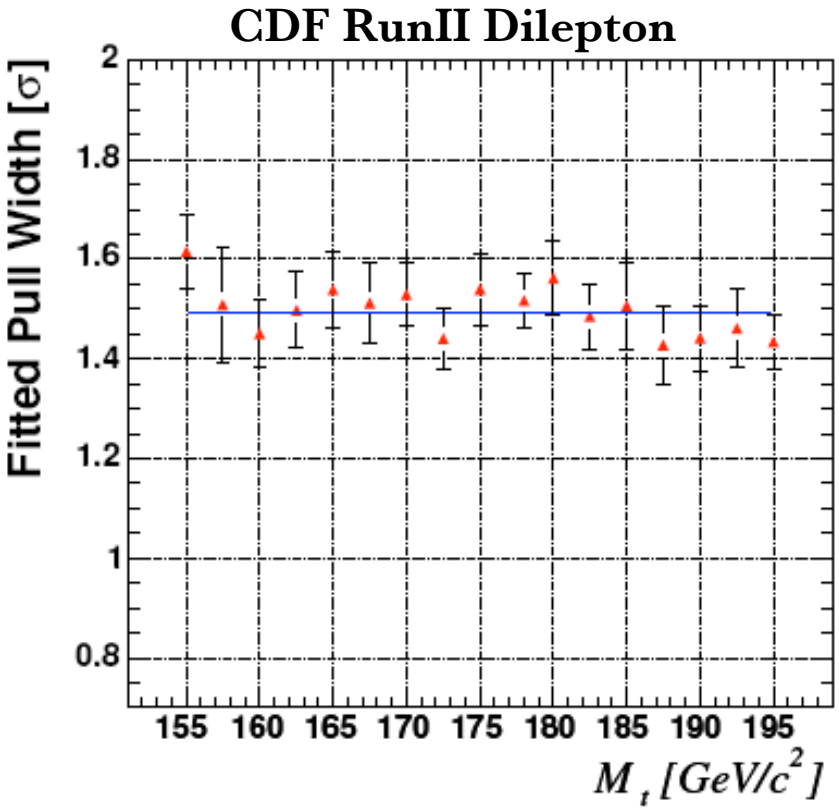
Integrate over 6 variables

2 Top masses
2 W masses
2 *b*-quark energies

CDF Dilepton Matrix Element: Simulation Calibration

Approximations in calculation require calibration in simulation.





Response

Linear.

Slope < 1 due to backgrounds.

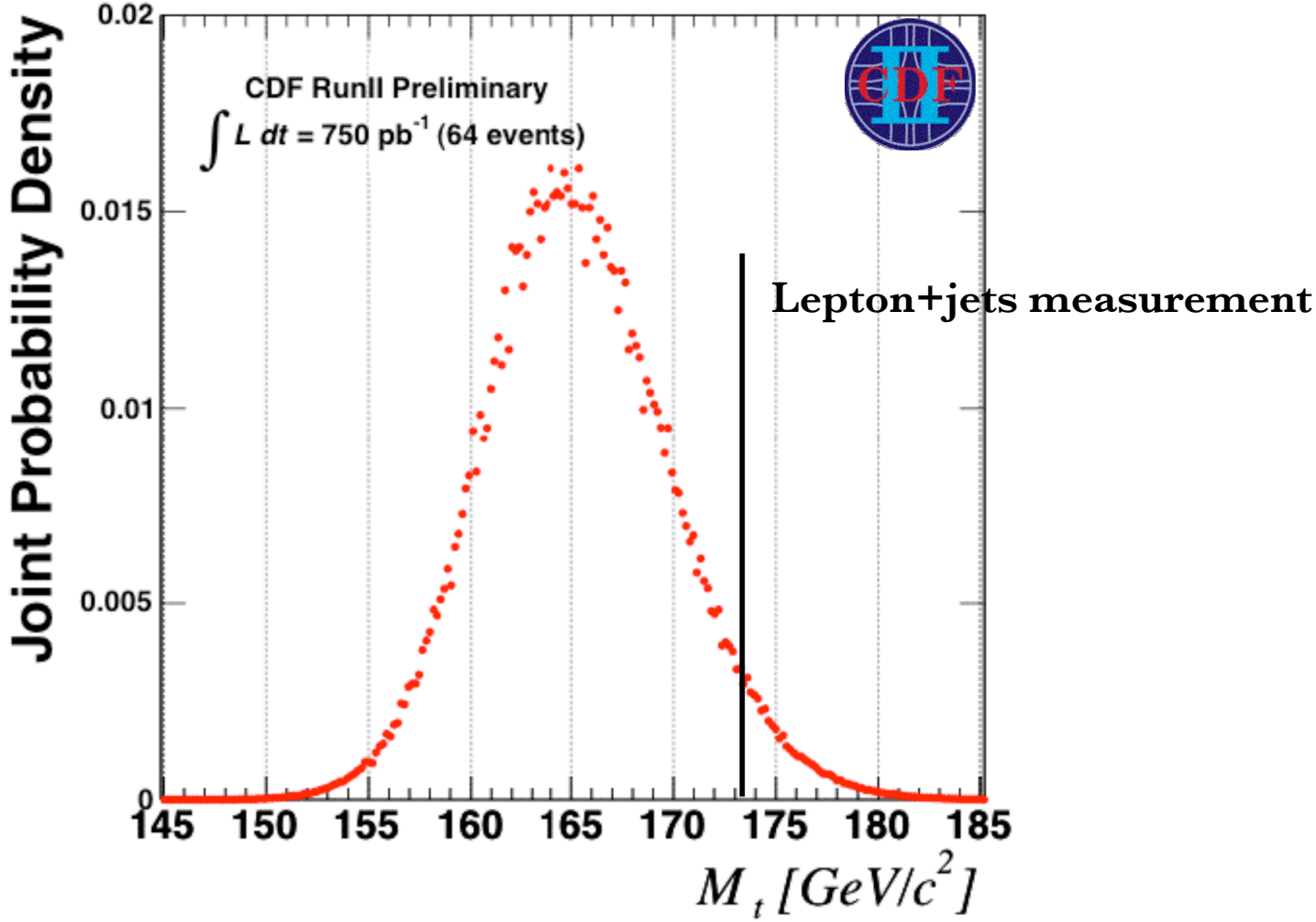
Pull width

Flat.

Pulls > 1 due to approximations made in P(Mt) calculation.

CDF Dilepton Matrix Element

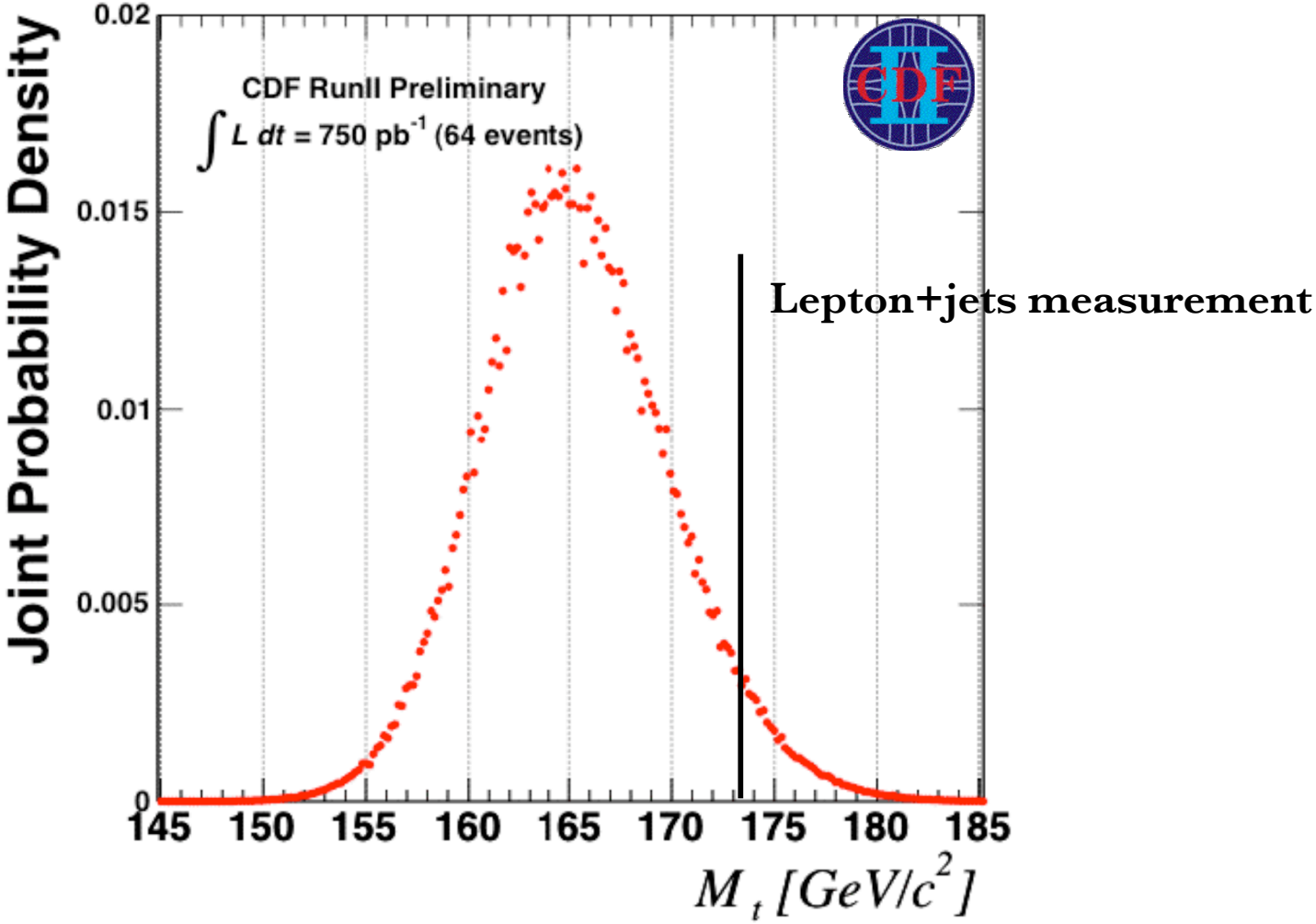
Most precise single dilepton measurement to date.



$$M_t = 164.6 \pm 4.5_{stat} \pm 3.1_{syst}\ \mathrm{GeV/c^2}$$

CDF Dilepton Matrix Element

Most precise single dilepton measurement to date.



$$M_t = 164.6 \pm 4.5_{stat} \pm 3.1_{syst}\ \mathrm{GeV/c^2}$$

$$b\text{-tagged: } M_t = 162.7 \pm 4.6_{stat} \pm 3.0_{syst}\ \mathrm{GeV/c^2}$$

CDF Dilepton Matrix Element: Systematic Errors

Source	$\Delta M_{top}(GeV/c^2)$
Jet Energy Scale	2.6
Background Statistics	0.8
Background Shape	0.8
ISR/FSR	0.7
Sample Composition	0.7
Parton Distributions	0.6
Generator	0.5
Method	0.3
Total	3.1

CDF Dilepton Matrix Element: Systematic Errors

Source	$\Delta M_{top}(GeV/c^2)$
Jet Energy Scale	2.6
Background Statistics	0.8
Background Shape	0.8
ISR/FSR	0.7
Sample Composition	0.7
Parton Distributions	0.6
Generator	0.5
Method	0.3
Total	3.1

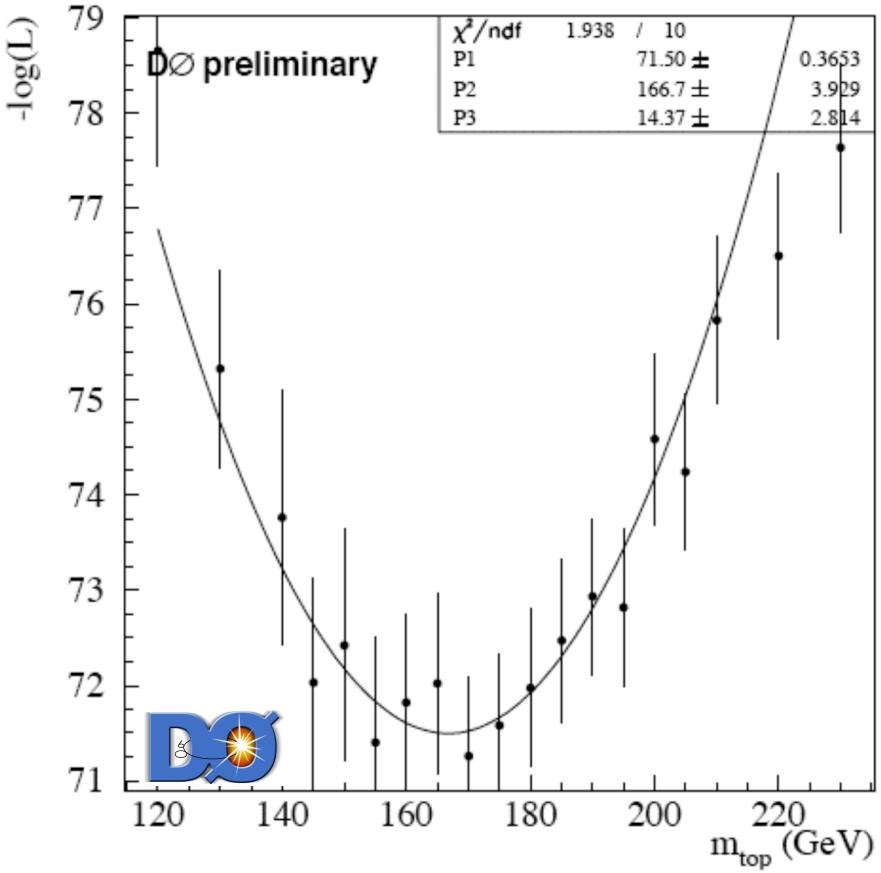
Could include $W \rightarrow jj$ calibration, but does not improve combined M_t.

Will include calibration from $Z \rightarrow bb$ and $\gamma + b$

D0 dilepton mass

(370 pb^{-1})

Pretag

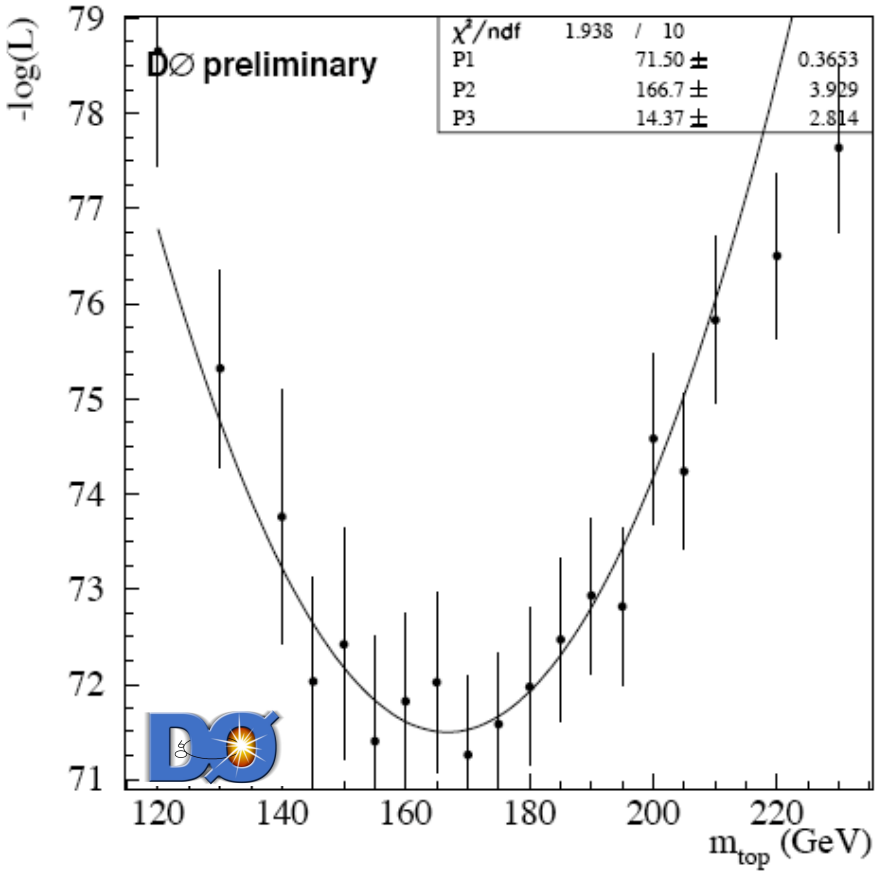


$M_t = 165.6 \pm 14_{stat} \pm 4_{syst}$ GeV/c^2

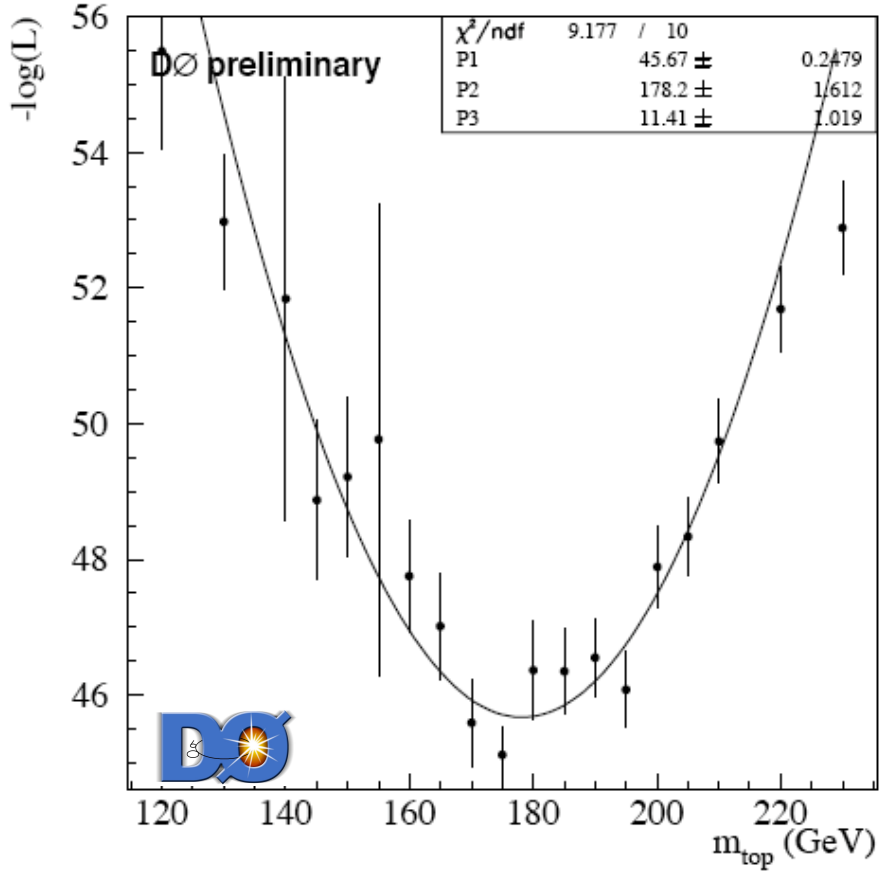
D0 dilepton mass

(370 pb^{-1})

Pretag



***b*-tagged**



$M_t = 165.6 \pm 14_{stat} \pm 4_{syst}$ GeV/c^2

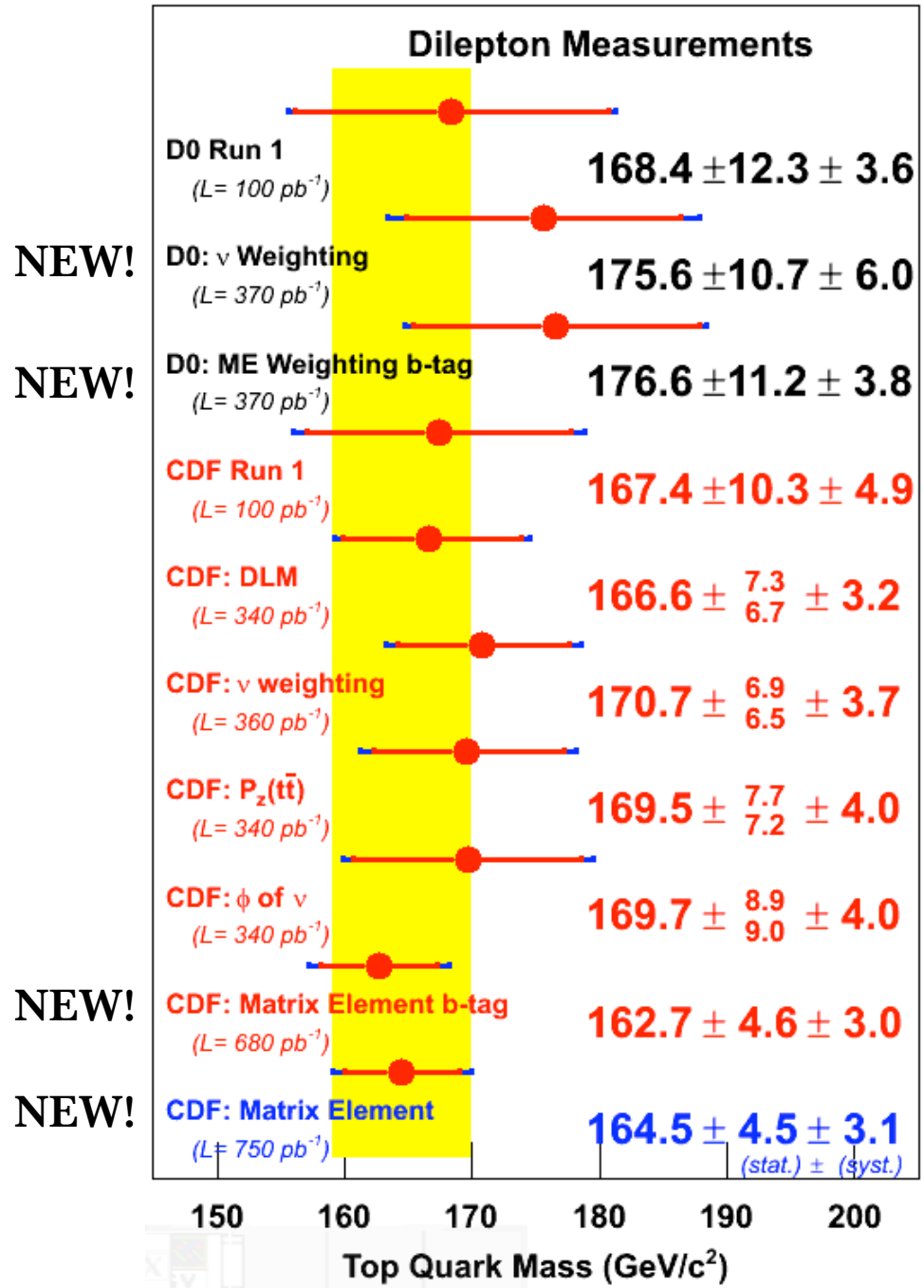
$M_t = 176.6 \pm 11_{stat} \pm 4_{syst}$ GeV/c^2

Dilepton Summary

Many methods being used

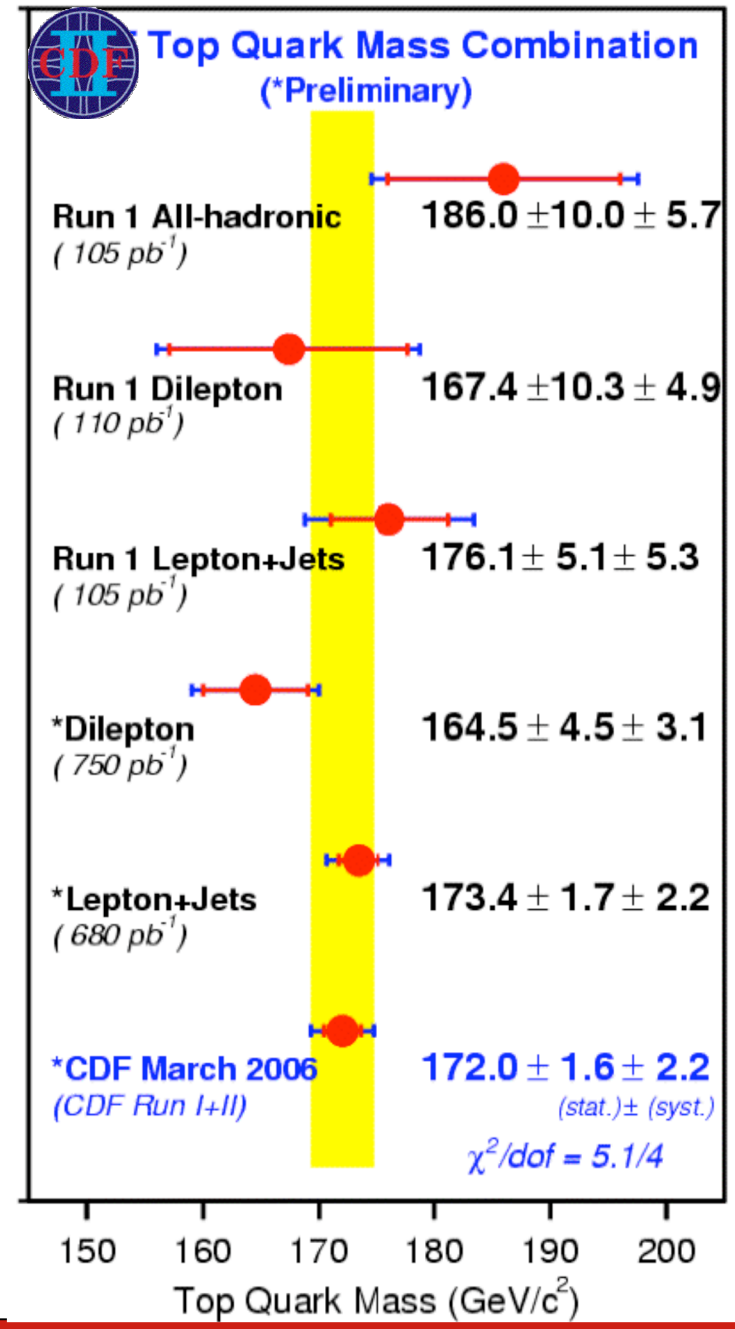
Differing systematic sensitivities

All measurements consistent.

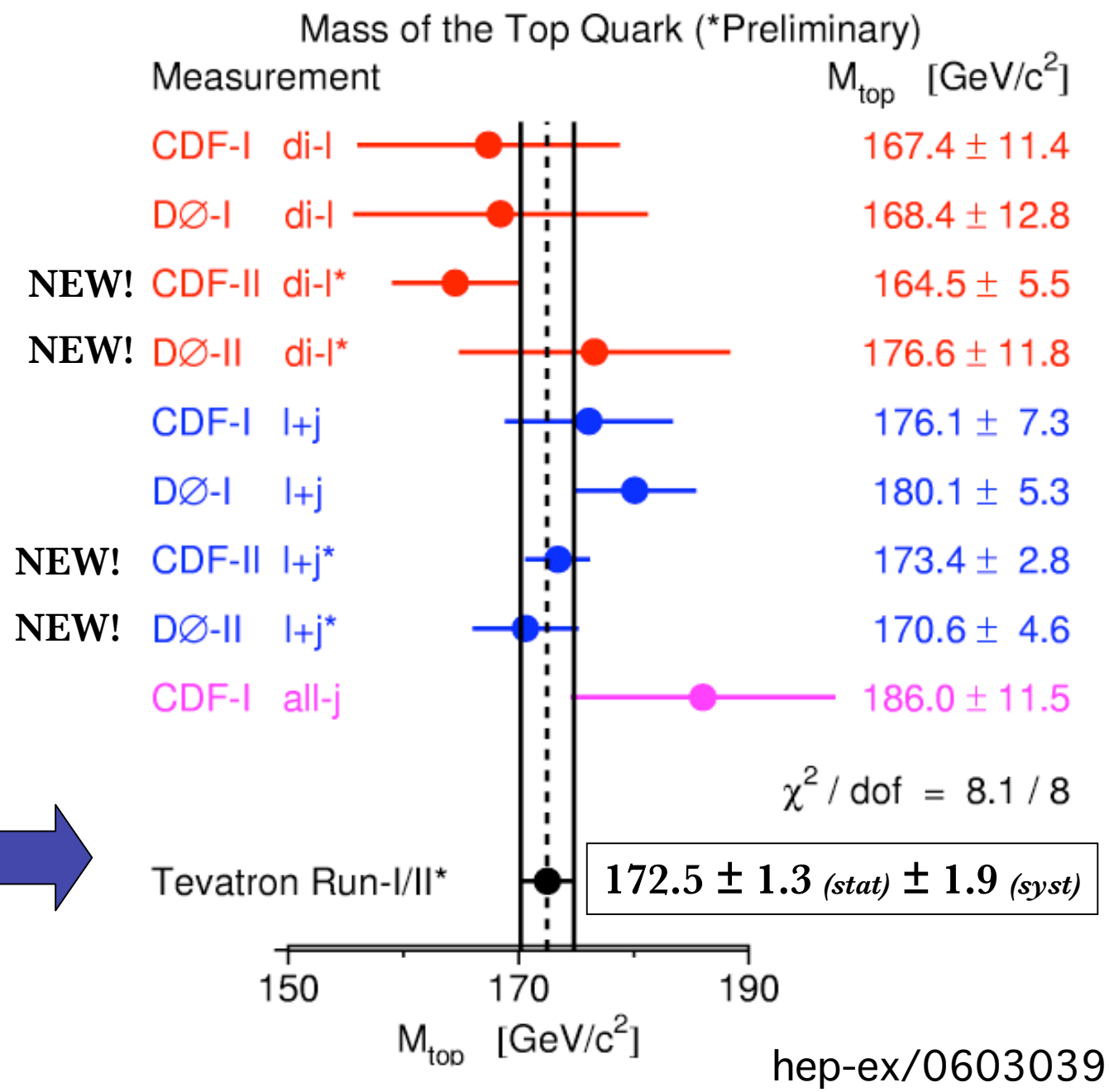


CDF Combination

CDF has combined its best measurements to give a total accuracy of 2.7 GeV.



World Average



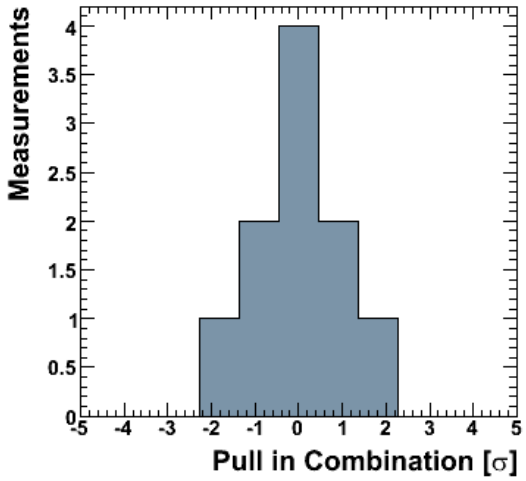
hep-ex/0603039

New World Average

Includes all updated measurements!

Error of 2.3 GeV, systematics limited.

World Average

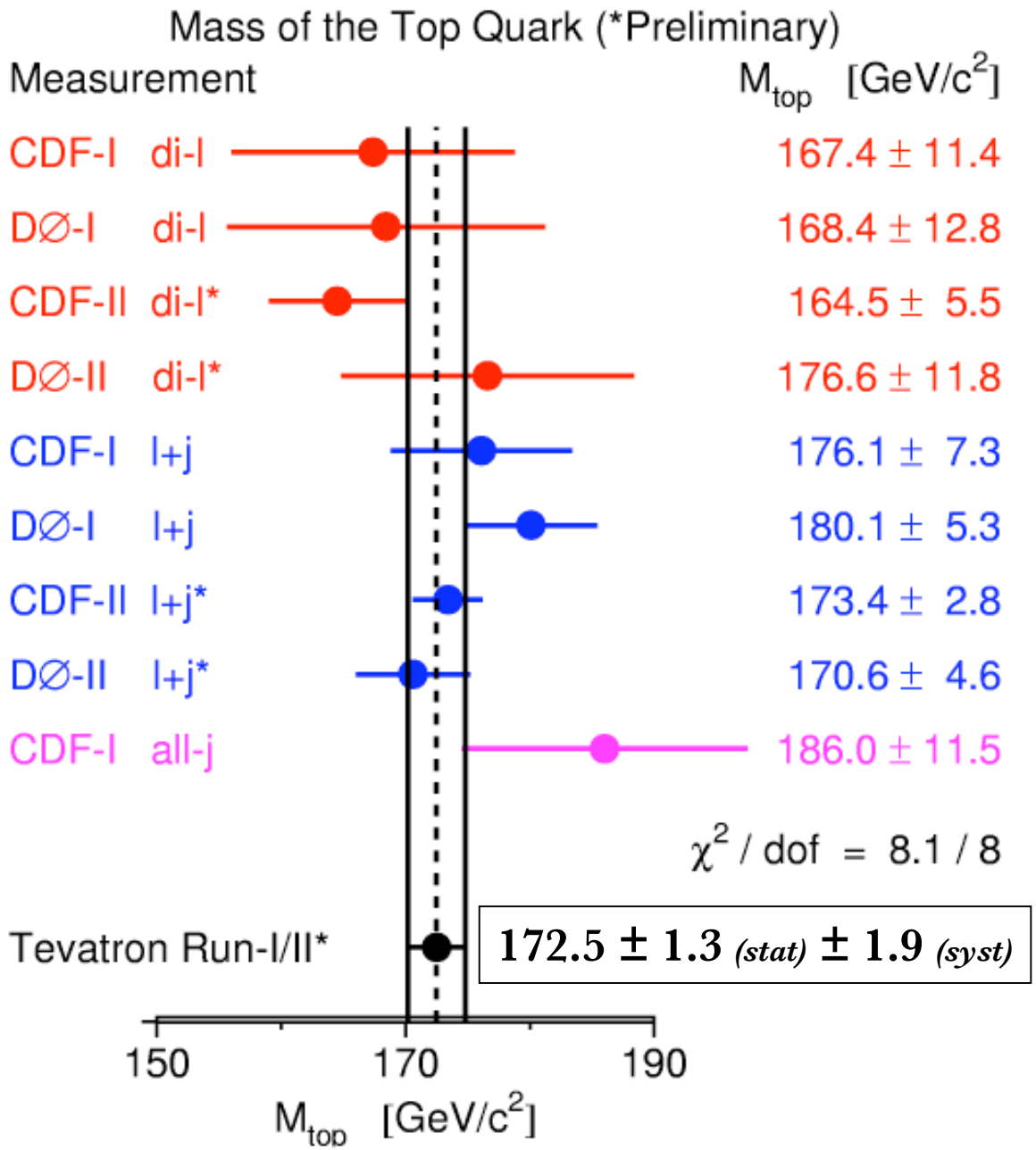


Total $\chi 2$ = 8.1 / 8 d.o.f.

New World Average

Includes all updated measurements!

Error of 2.3 GeV, systematics limited.



Channel Measurements

Comparison

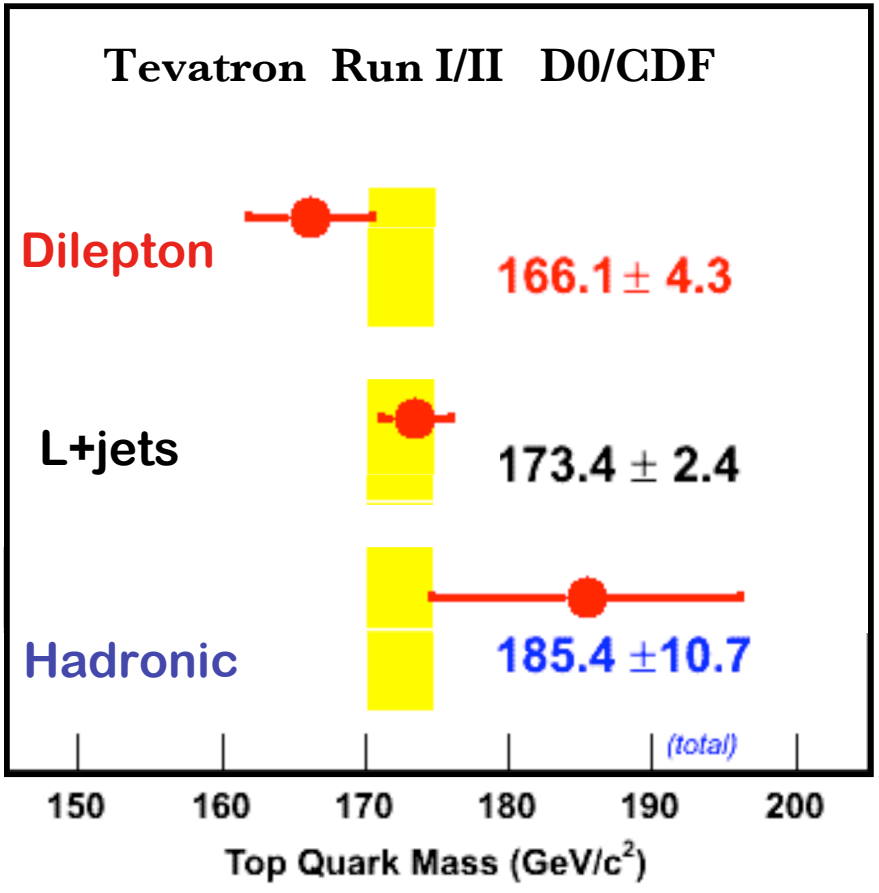
Improving precision of dilepton and hadronic measurements will allow for real comparison between channels.

Currently

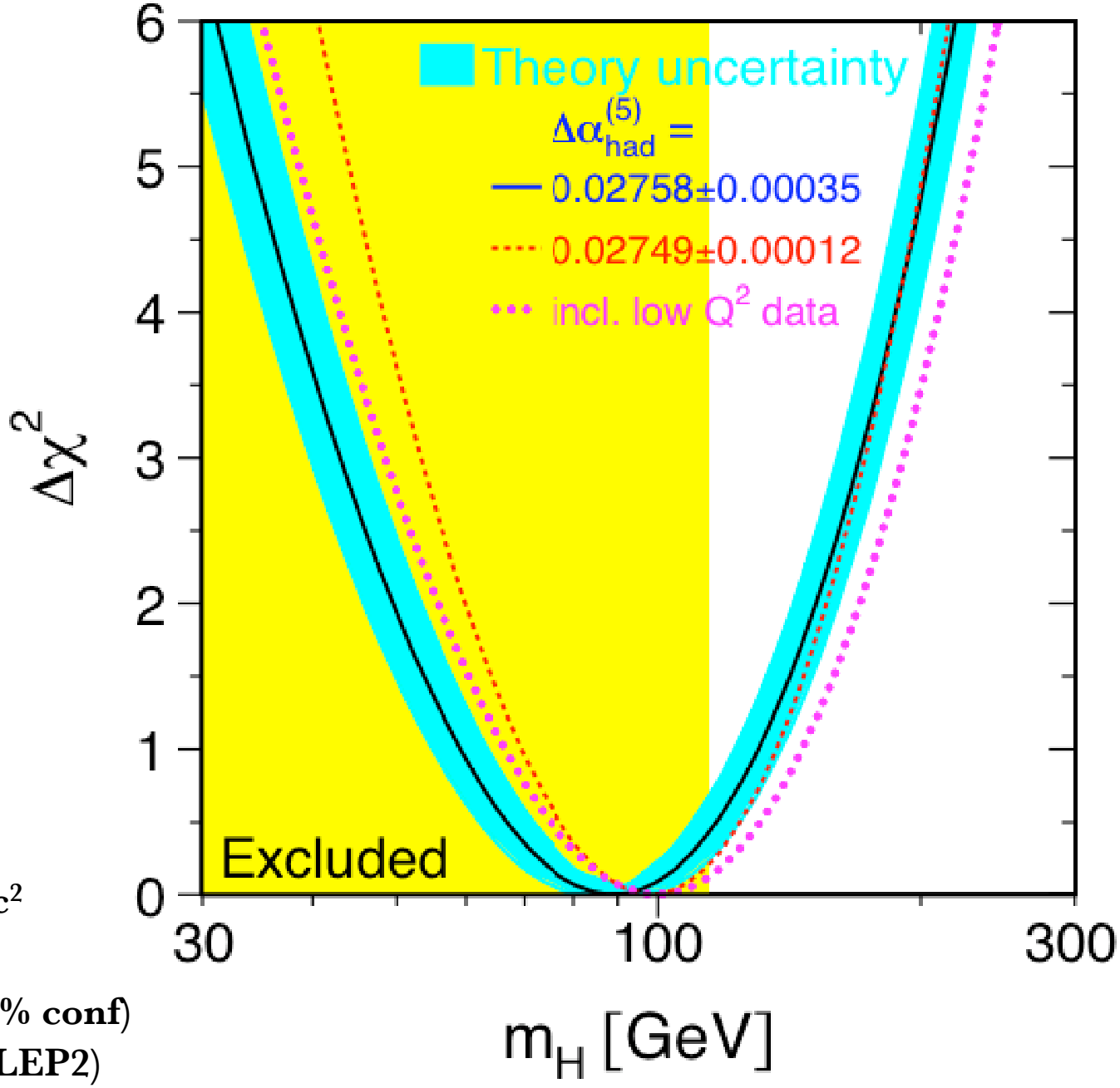
Dilepton channel appears low, but not statistically significant -- higher precision will reveal…

Future

High precision hadronic measurements coming soon.



Higgs Impact



M_{higgs} = **89 ± 36** GeV/c^2

M_{higgs} < **175** GeV/c^2 (95% conf)
(207 including LEP2)

Prospects for Systematics

Source	*dilepton* $\Delta M_{top}(GeV/c^2)$	*l+jets* $\Delta M_{top}(GeV/c^2)$
Jet Energy Scale	2.6	2.1
Background Statistics	0.8	0.3
Parton Distributions	0.6	0.3
Generator	0.5	0.2
Background Shape	0.8	0.5
ISR/FSR	0.7	0.6
Sample Composition	0.7	---
Method	0.3	---
B-tagging	---	0.1
Total	3.1	2.2

Key

Improves with luminosity

More sophisticated treatments ahead

Improves with CPU time

Summary

$$M_{top} = 172.5 \pm 1.3\ (stat) \pm 1.9\ (syst)$$

Precision Measurement

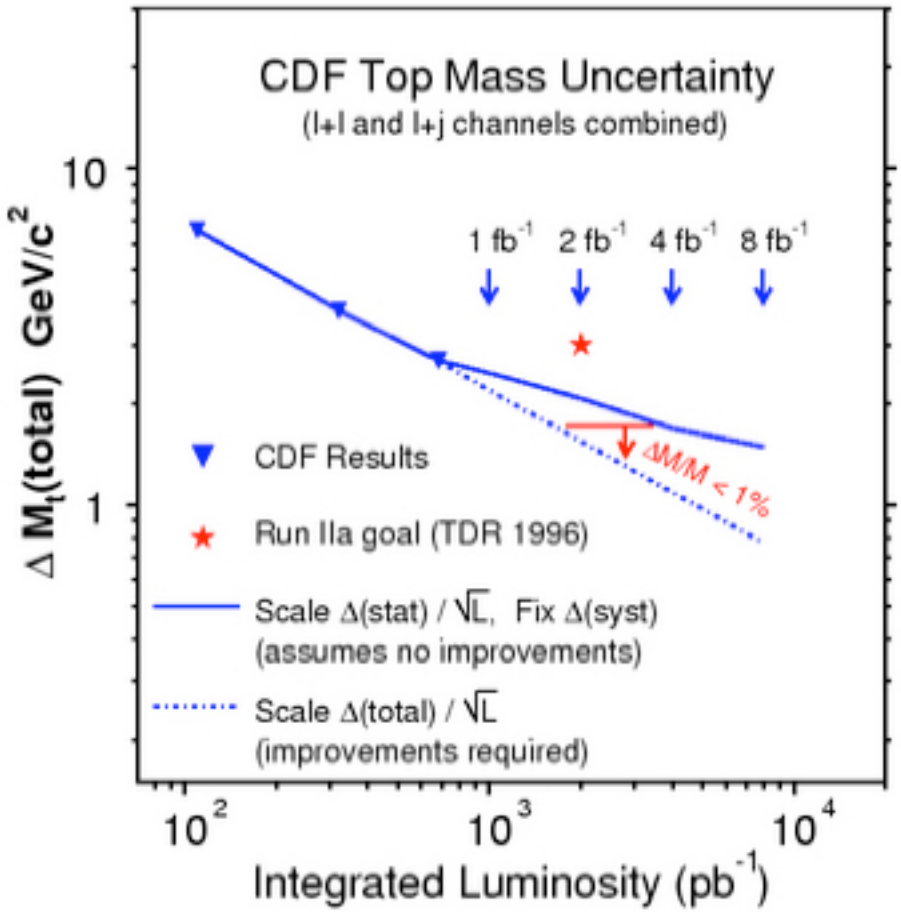
We have measured the top mass with an error of ~ 1.3%

Dileptons & l+jets consistent (so far!)

Room for Improvement

Systematic errors will decrease

All hadronic measurement in progress



Higher Precision Future

In 4-8fb^{-1},

- expect 1-2 GeV *total* error
- Dileptons become systematics limited.
- l+jets and dileptons approach equal weight
- Comparison of channels to test top hypothesis, look for new physics

- ***These results will be relevant for some years, even after LHC turn on***

World average

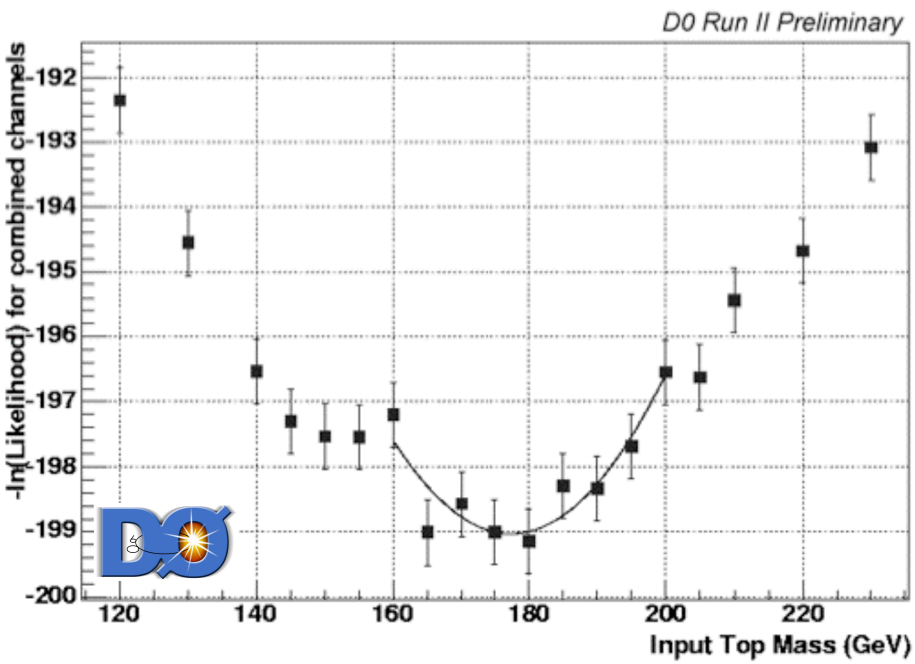
Category	Size [GeV/c2]
Statistical	1.3
Systematic	1.9
Jet Energy Scale	1.3
Radiation & PDFs	0.7
Background	0.5
Method	0.4
Monte Carlo	0.2
Uranium Noise & Multiple Int. [D0]	0.2
Total	2.3

$$M_{top} = 172.5 \pm 1.3\ (stat) \pm 1.9\ (syst)$$

Dilepton results

D0 ν Weighting Templates

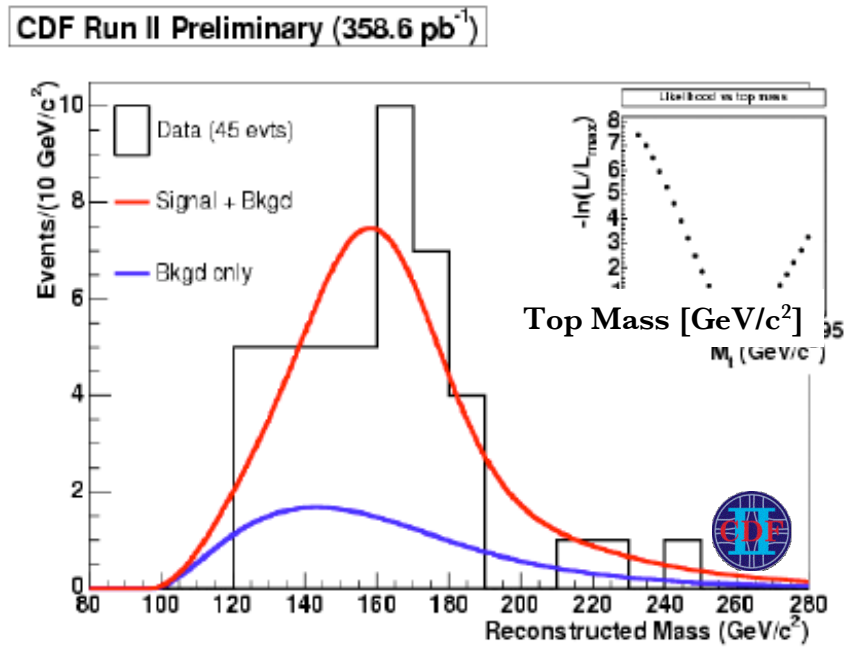
(370 pb^{-1})



$M_t = 175.6 \pm 11_{stat} \pm 6_{syst}$ GeV/c^2

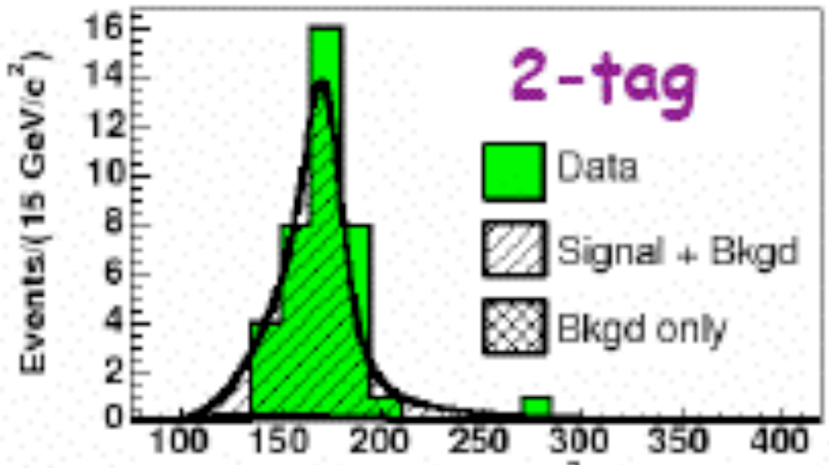
CDF ν Weighting Templates

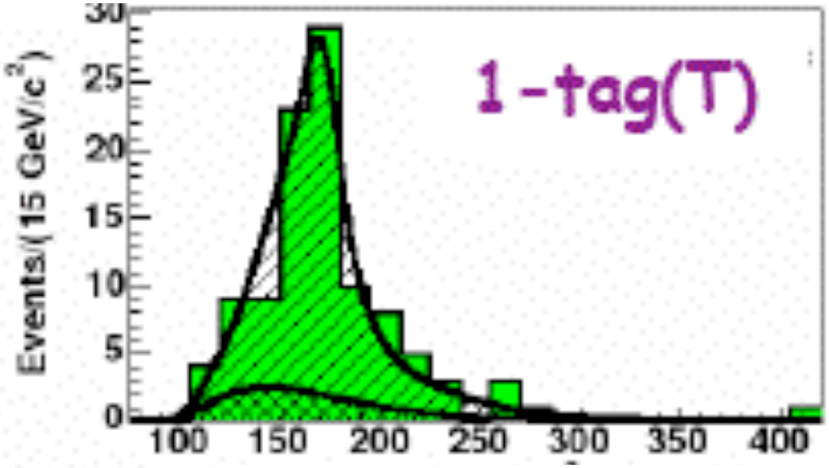
(360 pb^{-1})

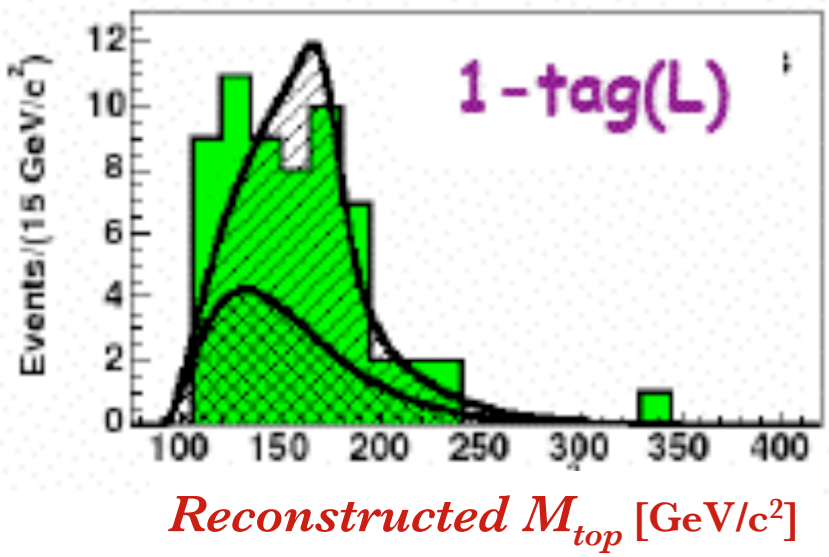


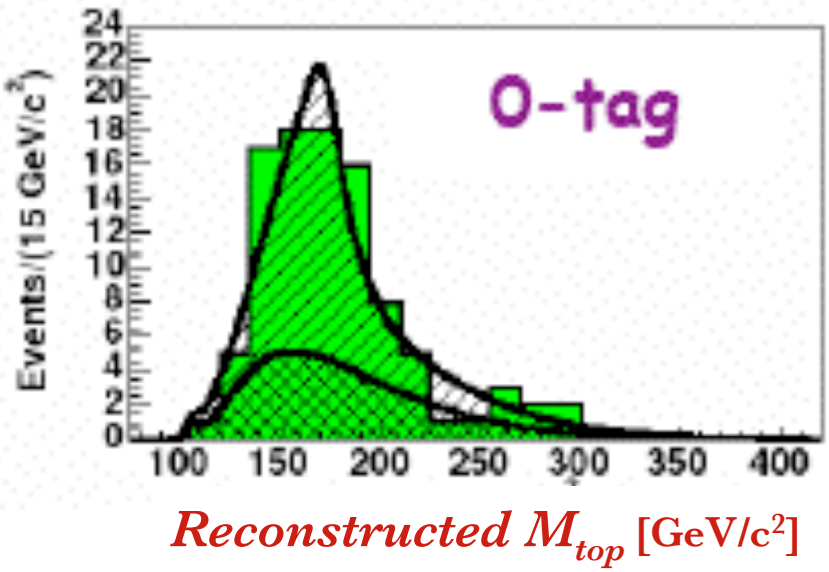
$M_t = 170.7 \pm 6.7_{stat} \pm 3.7_{syst}$ GeV/c^2

CDF Template results









Background Likelihood

We generalize the probability to be a weighted sum of signal & bg probabilities

$$P(\mathbf{x}|M_t) = P_s(\mathbf{x}|M_t)p_s + P_{bg1}(\mathbf{x})p_{bg1} + P_{bg2}(\mathbf{x})p_{bg2}\ldots$$

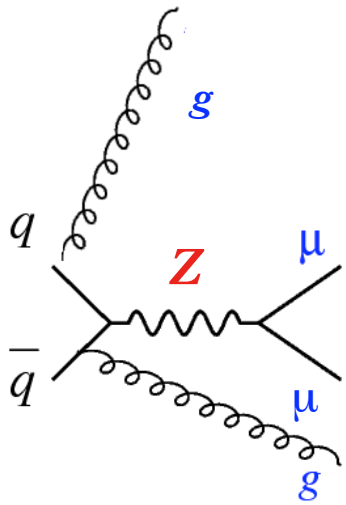
Where the weights are the expected sample fraction:

$$p_s(M_t) = \frac{\lambda_s(M_t)}{\lambda_s(M_t) + \lambda_b}$$

$$p_b = \frac{\lambda_b}{\lambda_s(M_t) + \lambda_b}$$

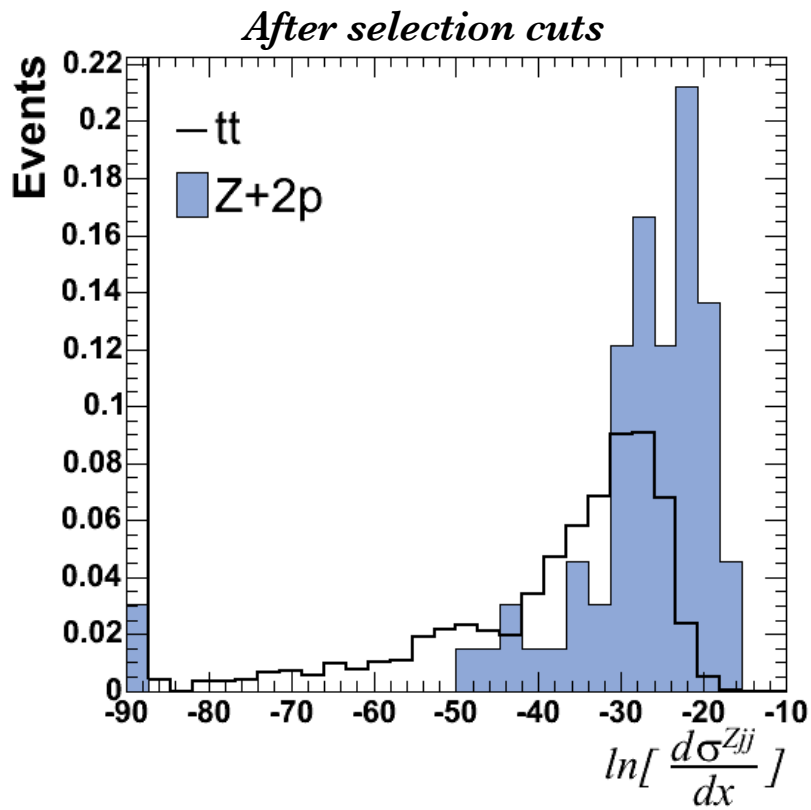
λ = expected number of events

Z+jets



0 unknowns

1 parton energy
1 parton energy
-2 *(P_T conservation)*



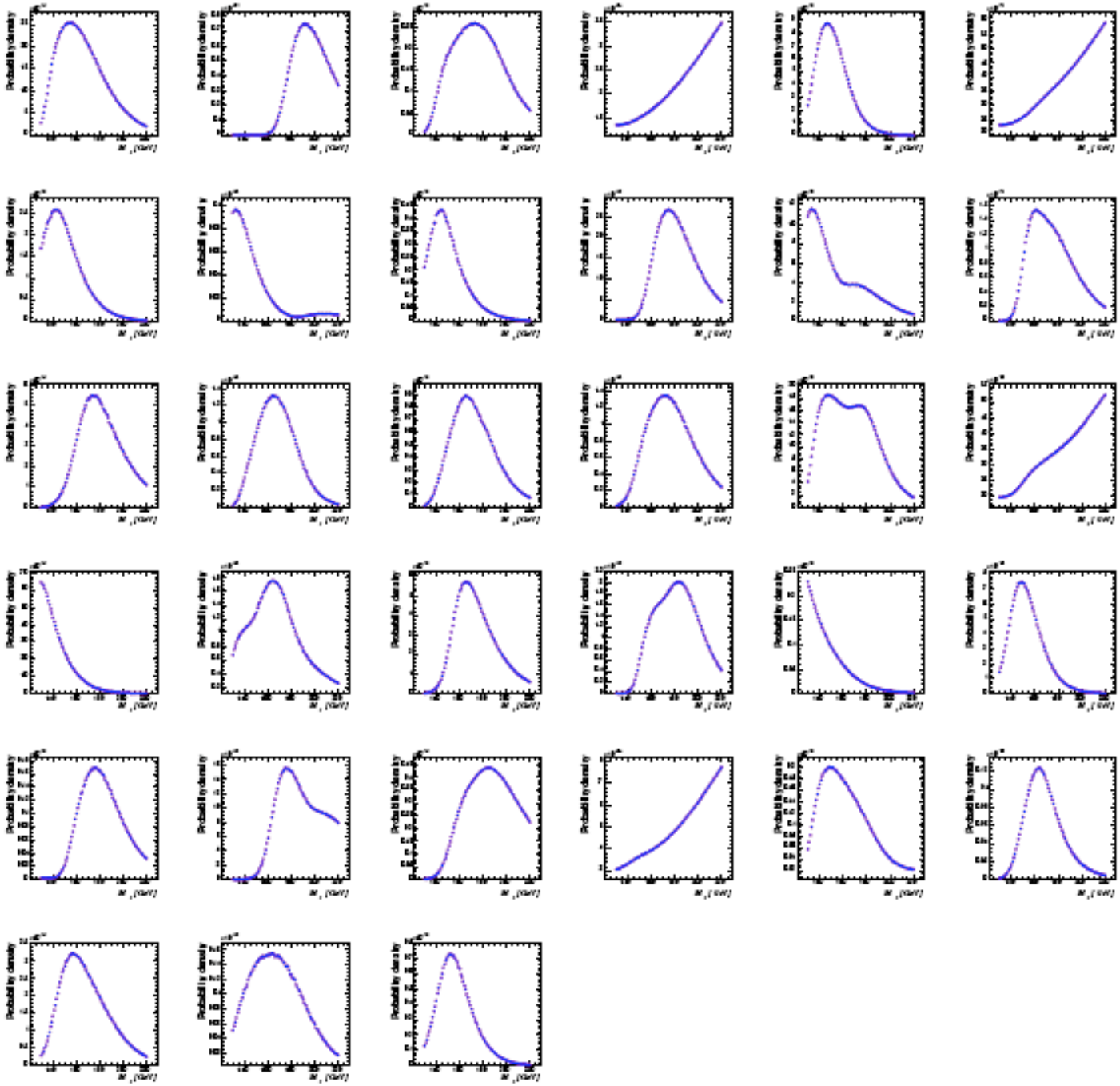
Matrix Element & Integrals

Add 2 integrals for P_T of Zjj system
Alpgen subroutine for Z+2p
Integration with Vegas

Data

33 candidates
signal and bg
probabilities

Range is
130-220 GeV/c^2

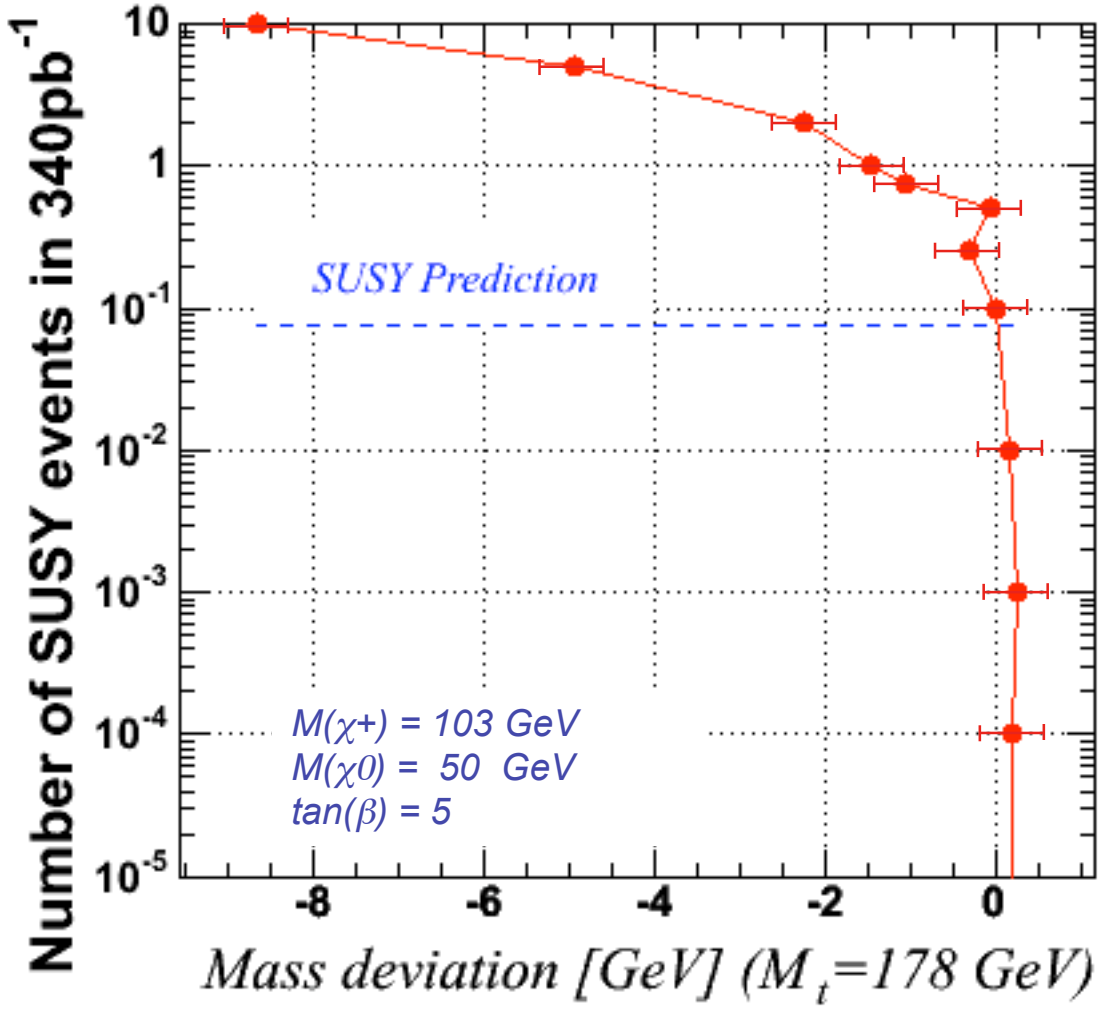


Effect of SUSY events on dilepton mass measurement

Chargino/Neutralino

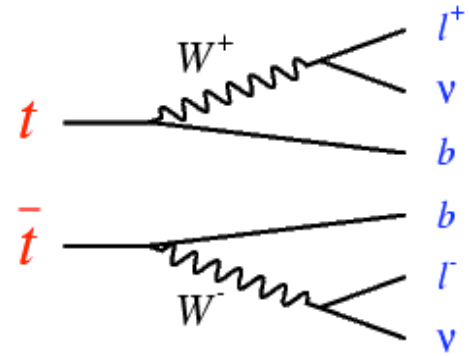
Topology is lll+2j or llqq

Msugra model
$M(\chi+)$ = 103 GeV
$M(\chi 0)$ = 50 GeV
$\tan(\beta)$ = 5
*σ*br = 150 fb*
Acc = 0.15%



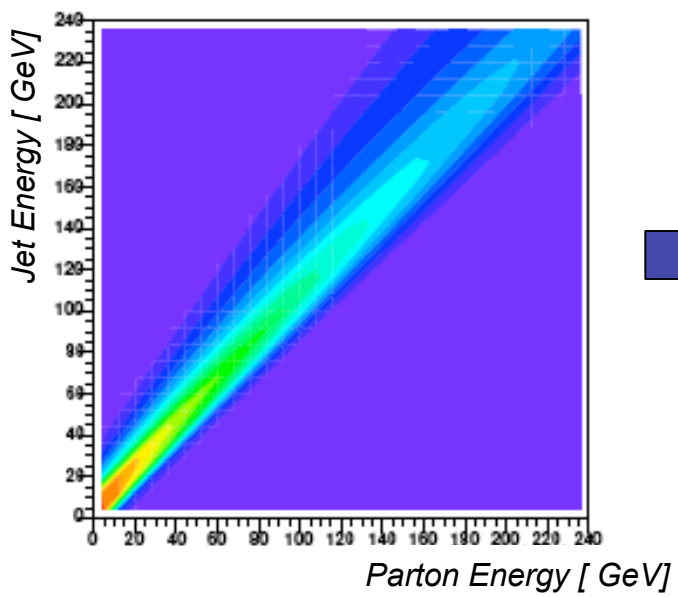
Direct Calculation

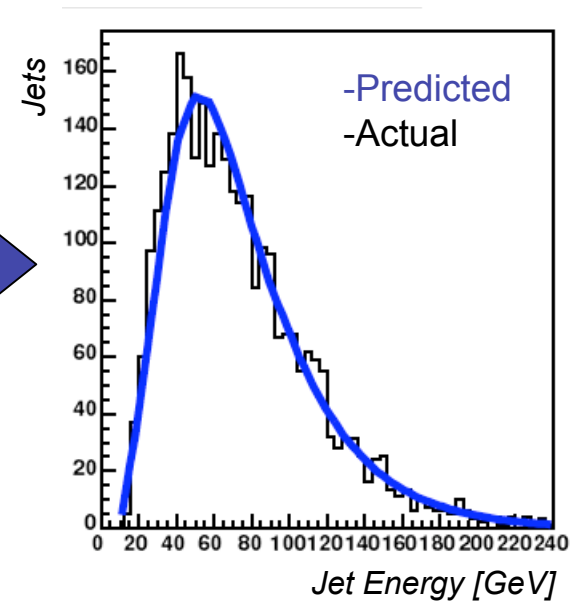
We know $P(\textit{partons} \mid M_t)$



We can parameterize parts of $P(\textit{event } x \mid \textit{partons})$:

For jets:





Assumptions

- Initial state
  - No initial state radiation
  - Transverse energy of system is negligible

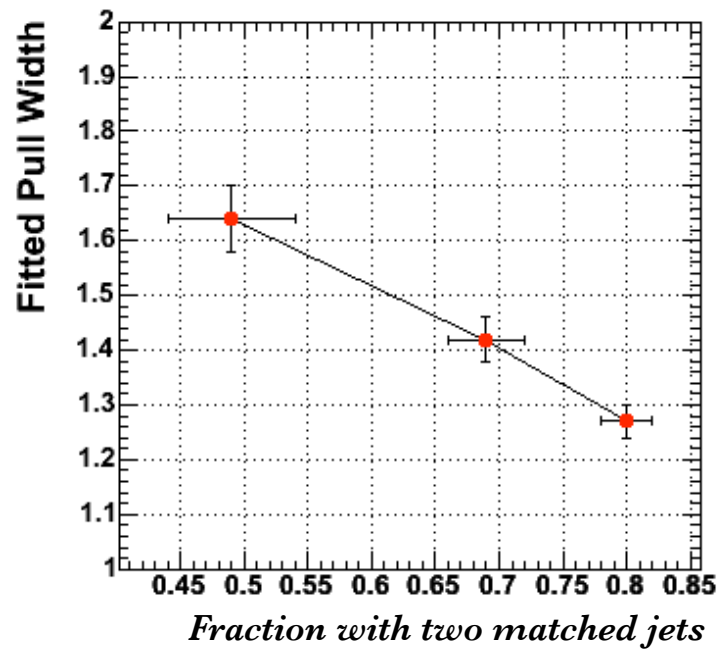
- Final state
  - Leptons
    - Energy well-measured
    - Direction well-measured

  - Jets
    - Jets arise from *b*-quarks
    - Direction well-measured
    - Energy can be parameterized from parton energy

- Assumptions make calculation tractable
  - balance sensitivity with computation time

Pull width

PW inflated because probability contains assumptions broken by a subset of events.



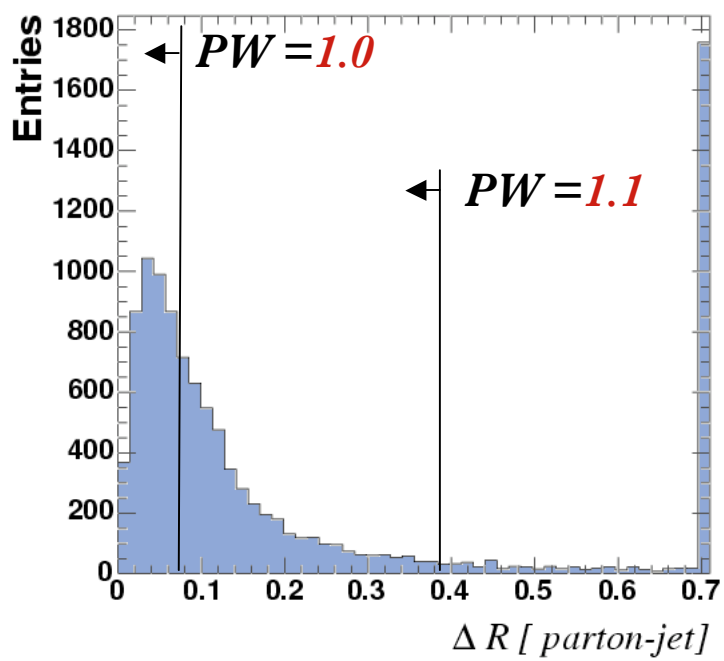
Assumptions held

Simple simulation: *width = 1.0*

Assemptions broken

Full simulation : *width = 1.4*

+ Two matched jets: *width* ➡ *1.2*
  + Well measured leptons: *width* ➡ *1.1*
    + Small parton-jet angle: *width* ➡ *1.0*



Scale factor for error

Flat in top mass
Flat in measured statistical error
Insensitive to systematic variations
Error on scale factor is ~ 0.03

Calculation

p 4-momentum of final partons
q 4-momentum of initial partons
x measured event variables

***For each event**, calculate differential cross-section:*

$$P(\mathbf{x}|M_t) = \frac{1}{N}\int d\Phi_6 |\mathcal{M}_{t\bar{t}}(p;M_t)|^2 \prod_{jets} f(p_i,\mathbf{x})\, f_{PDF}(q_1) f_{PDF}(q_2)$$

Phase-space Integral

Matrix Element

Transfer Functions

Only partial information available

Fix measured quantities

Integrate over 6 unmeasured parton quantities consistent with tt production and measured event.

CDF Lepton+Jets: Systematics

Systematics are largely due to uncertainties in modeling.

Source	$\Delta M_{top}(GeV/c^2)$
Residual Jet Energy Scale	0.7
ISR/FSR	0.6
B-jet modeling	0.6
Background Shape	0.5
Background JES	0.4
Parton Distributions	0.3
Generator	0.2
Simulation Statistics	0.3
B-tagging	0.1
Total	1.3

Uncertanties from semi-leptonic decay, fragmentation and color flow.

CDF Dilepton Matrix Element: Systematic Errors

Source	$\Delta M_{top}(GeV/c^2)$
Jet Energy Scale	2.6
Background Statistics	0.8
Background Shape	0.8
ISR/FSR	0.7
Sample Composition	0.7
Parton Distributions	0.6
Generator	0.5
Method	0.3
Total	3.1

Driven by small sample of mis-ID lepton candidates drawn from data.